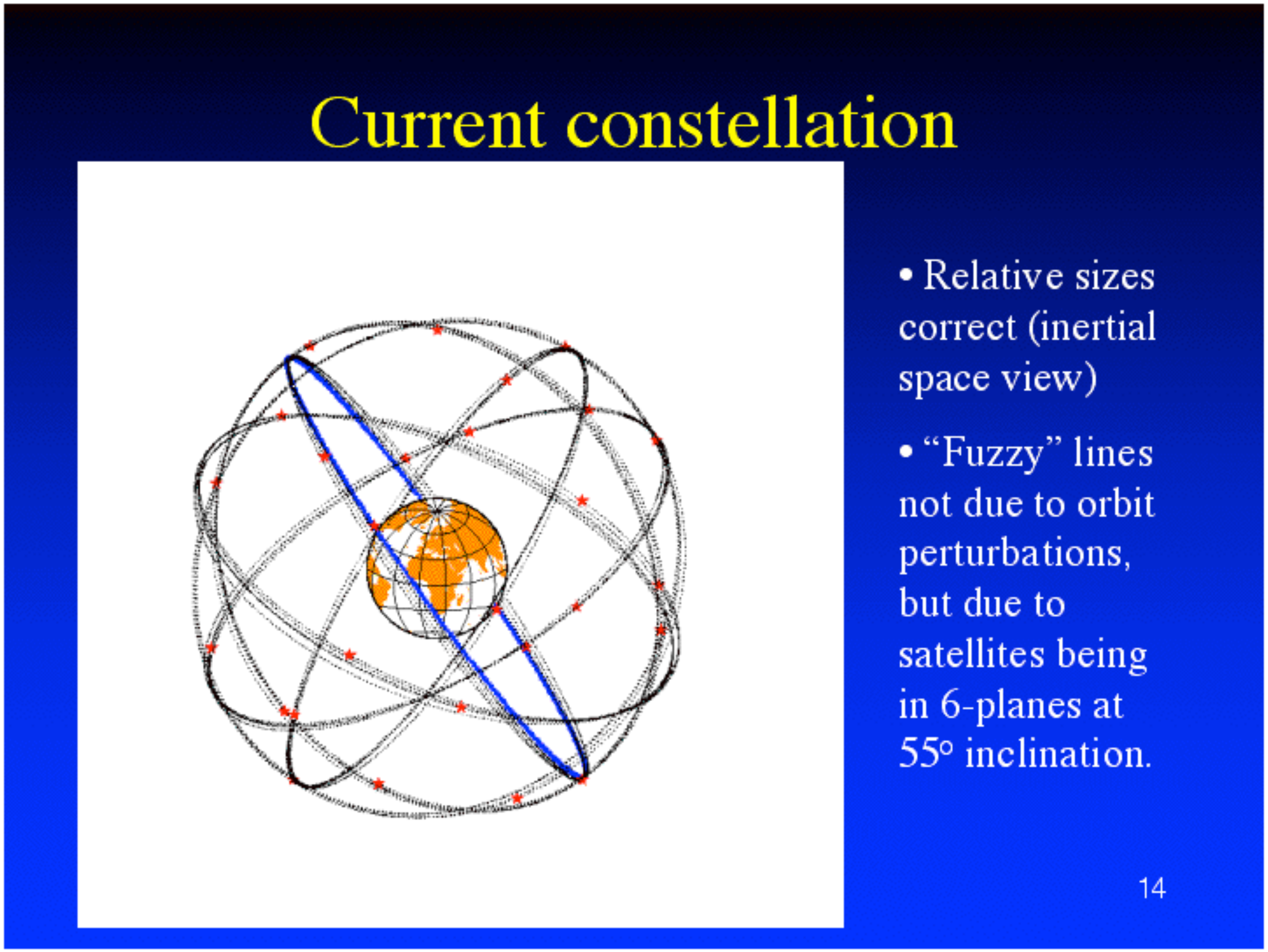
Fundamentals of GPS



22 April 2013

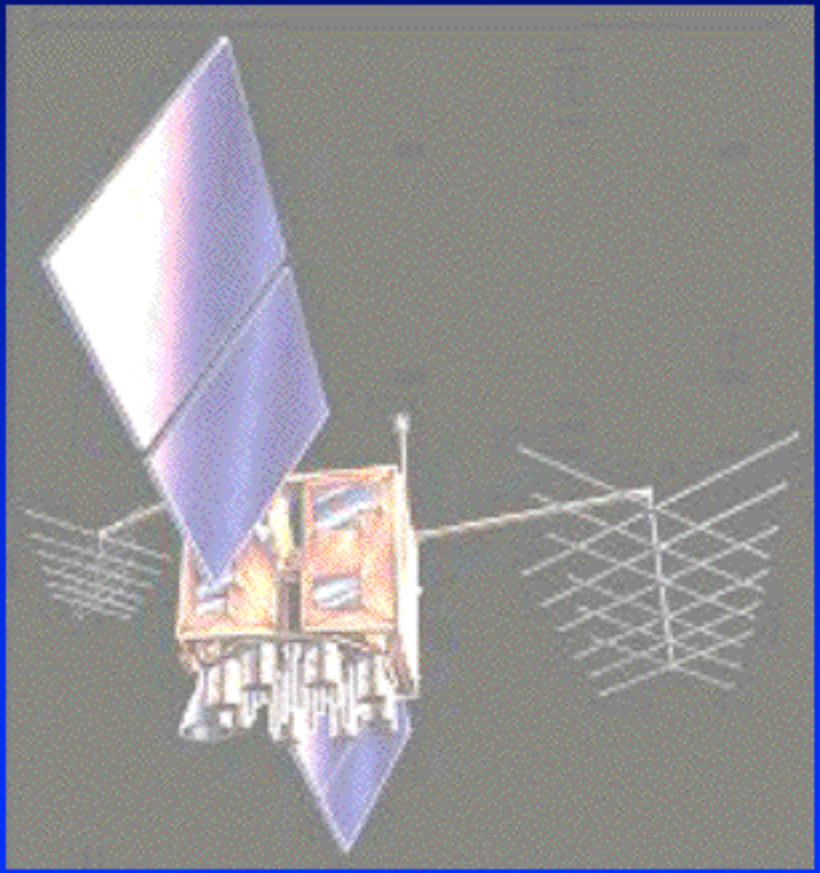
Resources

- http://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-215-modern-navigation-fall-2006/lecture-notes/12_215_lec16.pdf
- http://gge.unb.ca/Pubs/LN58.pdf
- http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap6/chap6.htm
- http://www.colorado.edu/geography/gcraft/notes/gps/gps_f.html

GPS Design

- Innovations:
  - Use multiple satellites (originally 21, now ~28)
  - All satellites transmit at same frequency
  - Signals encoded with unique "bi-phase, quadrature code" generated by pseudo-random sequence (designated by PRN, PR number): Spread-spectrum transmission.
  - Dual frequency band transmission:
    - L1 ~1.575 GHz, L2 ~1.227 GHz
    - Corresponding wavelengths are 190 mm and 244 mm

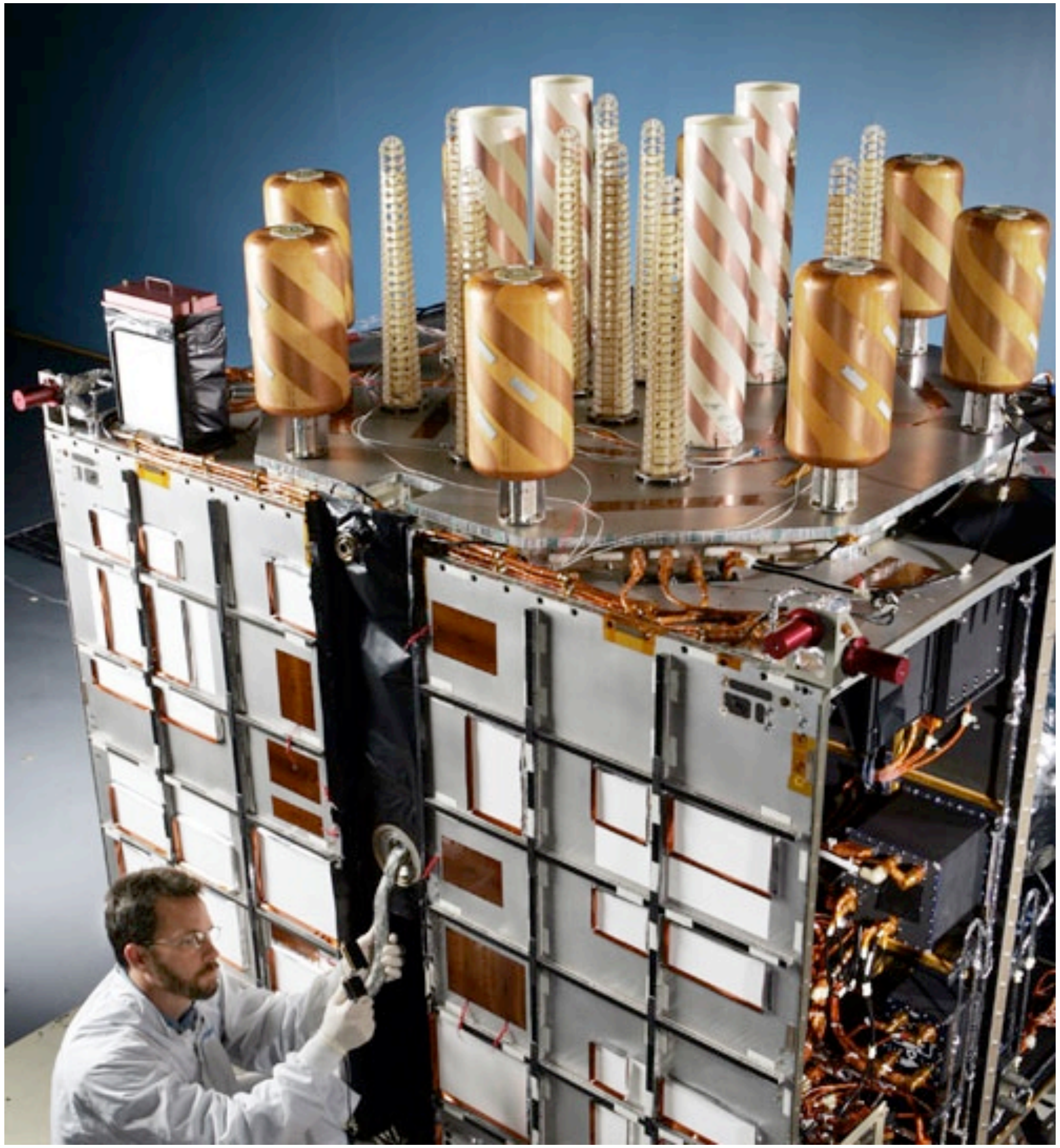
Latest Block IIR satellite (1,100 kg)

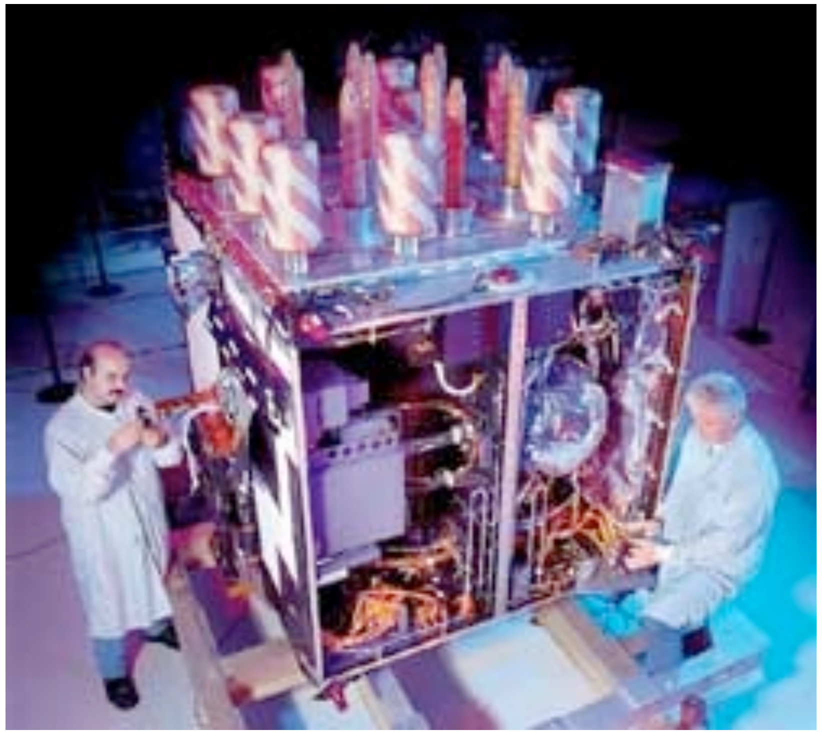


° Transmission array is made up of 12 helical antenna in two rings of 43.8 cm (8 antennas) and 16,2 cm (4 antennas) radii

° Total diameter is 87 cm

° Solar panels lead to large solar radiation pressure effects.





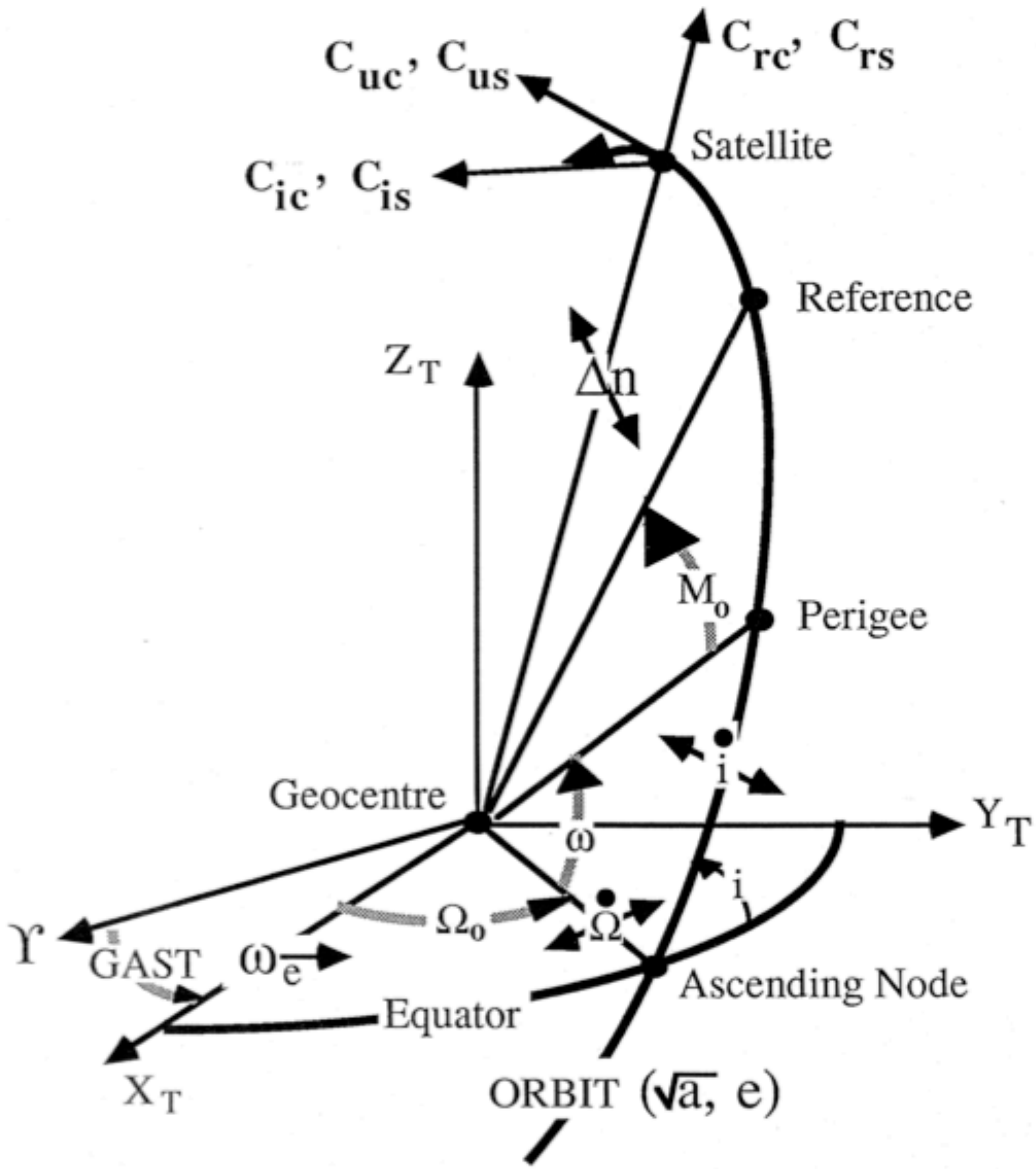
```
     2.10           N: GPS NAV DATA                         RINEX VERSION / TYPE
teqc  2013Mar15     CORS-ADM Account    20130414 23:45:05UTCPGM / RUN BY / DATE
Solaris x86 5.10|AMD64|cc SC5.8 -xarch=amd64|=+|=+          COMMENT
     2              NAVIGATION DATA                         COMMENT
RINGO/GLOBAL        NATIONAL GEODETIC SURVEY                COMMENT
    2.6077D-08  1.4901D-08 -1.1921D-07 -5.9605D-08          ION ALPHA
    1.2902D+05  1.6384D+04 -2.6214D+05  3.2768D+05          ION BETA
   -6.519258022308D-09-1.332267629550D-14   503808     1735 DELTA-UTC: A0,A1,T,W
                                                            END OF HEADER
 1 13  4  9 18  0  0.0 1.472141593690D-05 3.410605131650D-12 0.000000000000D+00
    2.400000000000D+01-5.400000000000D+01 4.381968121690D-09 2.058269675580D+00
   -2.793967723850D-06 1.720851752910D-03 1.030229032040D-05 5.153705976490D+03
    2.376000000000D+05-7.450580596920D-09-5.873366441290D-01-2.607703208920D-08
    9.604077583470D-01 1.776875000000D+02 3.090839083680D-01-8.000332840650D-09
   -4.764484362600D-10 1.000000000000D+00 1.735000000000D+03 0.000000000000D+00
    2.000000000000D+00 0.000000000000D+00 8.381903171540D-09 2.400000000000D+01
    0.000000000000D+00
 1 13  4  9 22  0  0.0 1.477124169470D-05 3.410605131650D-12 0.000000000000D+00
    4.100000000000D+01-4.075000000000D+01 4.279106846640D-09-2.123900168530D+00
   -2.061948180200D-06 1.721313572490D-03 1.082755625250D-05 5.153704549790D+03
    2.520000000000D+05-2.793967723850D-08-5.874506551860D-01 3.539025783540D-08
    9.604012205660D-01 1.710312500000D+02 3.083902306510D-01-7.871756579900D-09
   -5.368080979730D-10 1.000000000000D+00 1.735000000000D+03 0.000000000000D+00
    2.000000000000D+00 0.000000000000D+00 8.381903171540D-09 4.100000000000D+01
    2.592000000000D+05
 1 13  4 10  0  0  0.0 1.479592174290D-05 3.410605131650D-12 0.000000000000D+00
    4.200000000000D+01-3.984375000000D+01 4.367324724090D-09-1.074288840050D+00
   -2.112239599230D-06 1.721556764100D-03 1.055933535100D-05 5.153702756880D+03
    2.592000000000D+05 7.450580596920D-08-5.875068751280D-01-4.097819328310D-08
    9.603968888650D-01 1.785937500000D+02 3.089400011170D-01-7.911401311840D-09
   -6.403838015420D-10 1.000000000000D+00 1.735000000000D+03 0.000000000000D+00
    2.000000000000D+00 0.000000000000D+00 8.381903171540D-09 4.200000000000D+01
    2.592000000000D+05
```

BROADCAST EPHEMERIS MESSAGE PARAMETERS

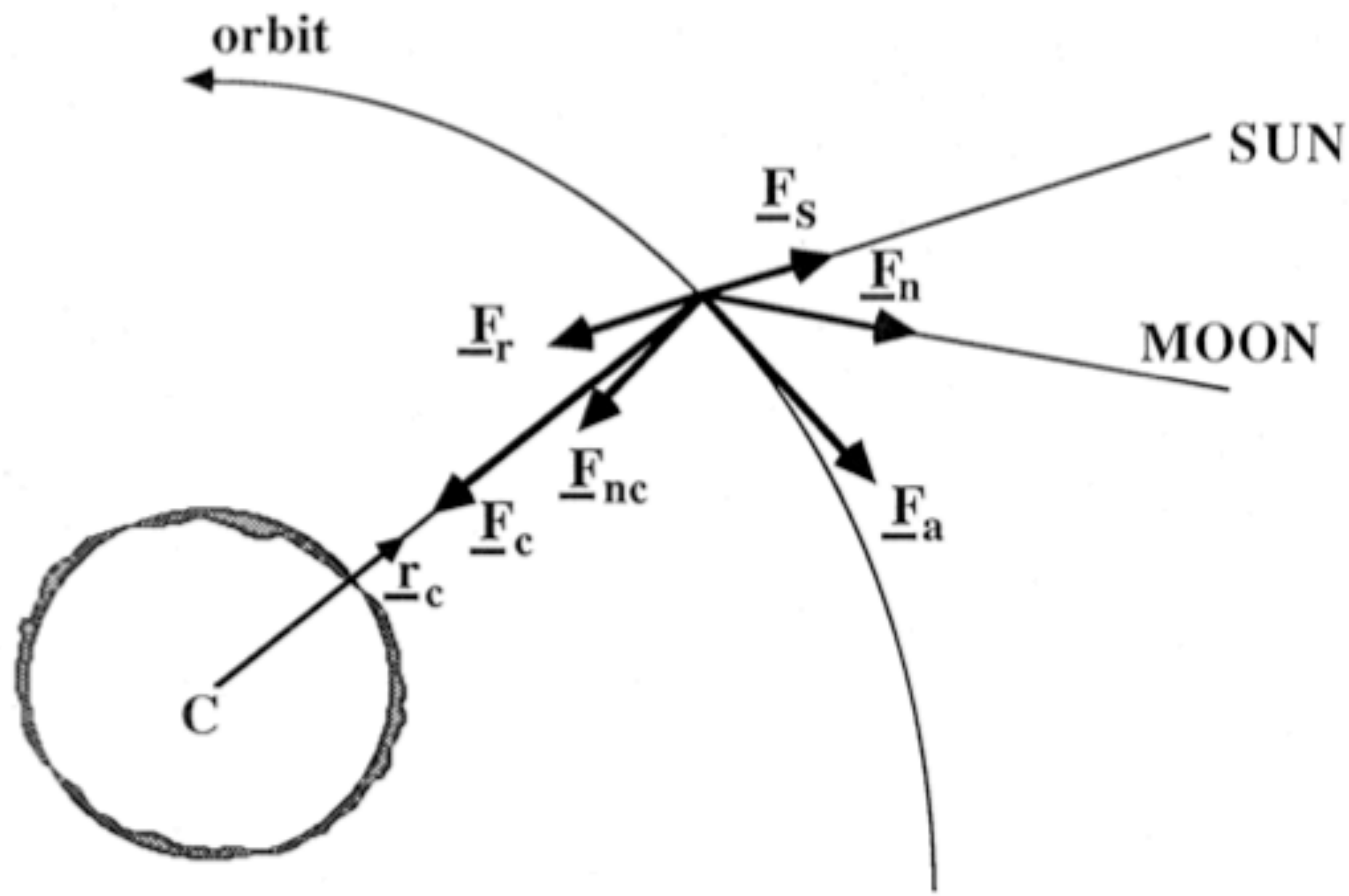
M_0	Mean anomaly
Δn	Mean motion difference
e	Eccentricity
$\sqrt{a}$	$\sqrt{}$ of semi-major axis
Ω_0	Right ascension parameter
i_0	Inclination at reference time t_{oe}
ω	Argument of perigee
$\dot{\Omega}$	Rate of right ascension
$\dot{i}$	Rate of inclination
C_{uc}, C_{us}	Corrections to argument of latitude
C_{rc}, C_{rs}	Corrections to orbital radius
C_{ic}, C_{is}	Corrections to inclination
t_{oe}	Ephemeris reference time

GPS ORBIT DESCRIPTION

- 16 PARAMETERS
- NEW VALUES EACH HOUR

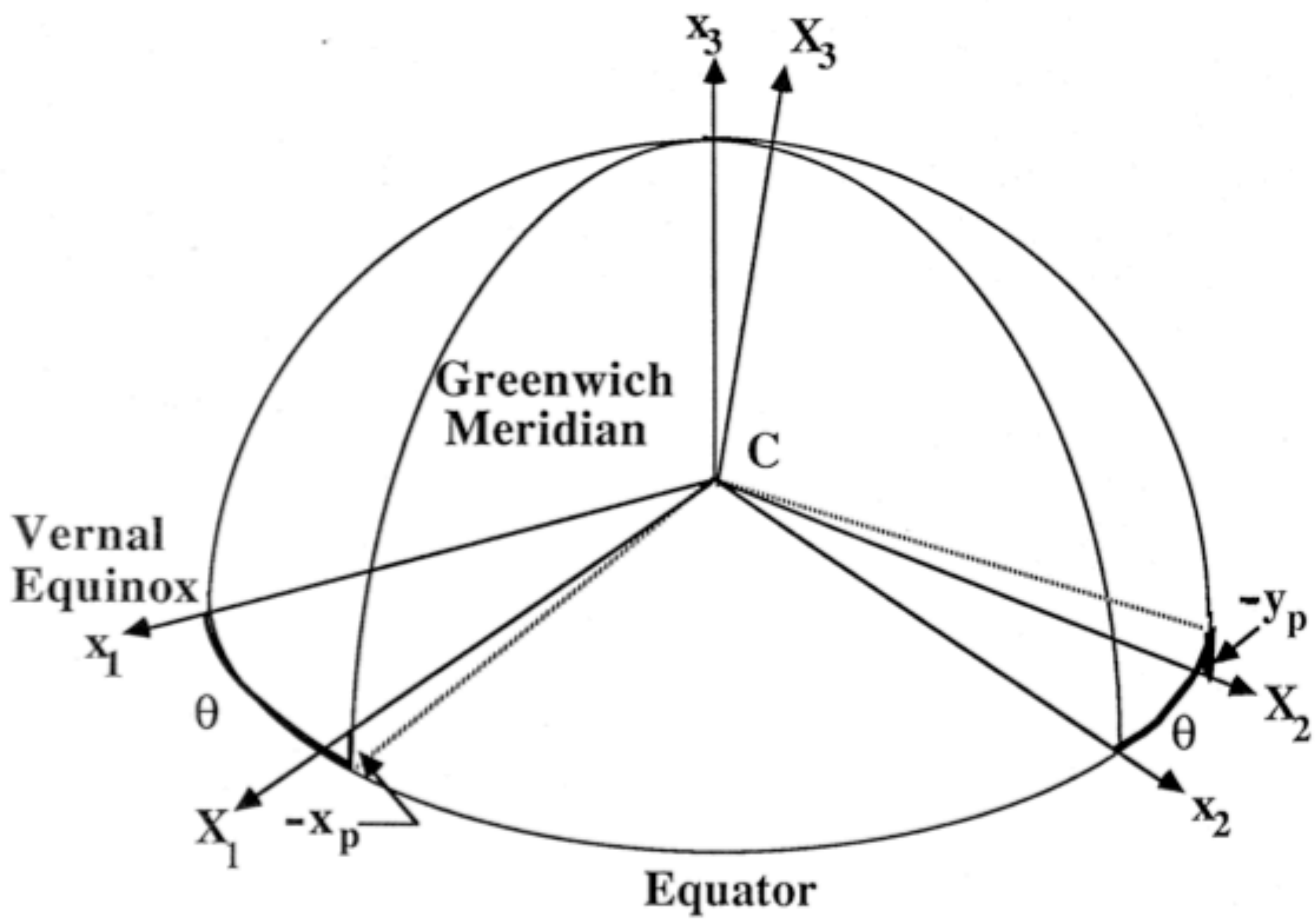


FORCES ACTING ON SATELLITES



Important for GPS satellites	• $\underline{F}_c$	Geocentric gravitational attraction of the earth
	• $\underline{F}_{nc}$	Non-central gravitational attraction of the earth
	• $\underline{F}_n, \underline{F}_s$	Third body effects of the moon and the sun
	• $\underline{F}_r$	Solar radiation pressure
Other forces	• $\underline{F}_a$	Atmospheric drag force
		Tidal forces (not shown)
		Magnetic forces (not shown)

COORDINATE SYSTEMS



Satellite motion is described in RA-system (x_1, x_2, x_3)

Terrestrial positioning refers to CT-system (X_1, X_2, X_3)

The two are related by :

Greenwich apparent sidereal time (θ)

Polar motion components (x_p, y_p)

TABLE A3
GPS NAVIGATION MESSAGE FILE - HEADER SECTION DESCRIPTION

	HEADER LABEL (Columns 61-80)	DESCRIPTION	FORMAT	
	RINEX VERSION / TYPE	- Format version (2.11) - File type ('N' for Navigation data)	F9.2,11X, A1,19X	
	PGM / RUN BY / DATE	- Name of program creating current file - Name of agency creating current file - Date of file creation	A20, A20, A20	
*	COMMENT	Comment line(s)	A60	*
*	ION ALPHA	Ionosphere parameters A0-A3 of almanac (page 18 of subframe 4)	2X,4D12.4	*
*	ION BETA	Ionosphere parameters B0-B3 of almanac	2X,4D12.4	*
*	DELTA-UTC: A0,A1,T,W	Almanac parameters to compute time in UTC (page 18 of subframe 4) A0,A1: terms of polynomial T : reference time for UTC data W : UTC reference week number. Continuous number, not mod(1024)!	3X,2D19.12, 2I9 *)	*
*	LEAP SECONDS	Delta time due to leap seconds	I6	*
	END OF HEADER	Last record in the header section.	60X	

Records marked with * are optional

GPS NAVIGATION MESSAGE FILE - DATA RECORD DESCRIPTION

OBS. RECORD	DESCRIPTION	FORMAT
PRN / EPOCH / SV CLK	- Satellite PRN number - Epoch: Toc - Time of Clock year (2 digits, padded with 0 if necessary) month day hour minute second - SV clock bias (seconds) - SV clock drift (sec/sec) - SV clock drift rate (sec/sec2)	I2, 1X,I2.2, 1X,I2, 1X,I2, 1X,I2, 1X,I2, F5.1, 3D19.12 *)
BROADCAST ORBIT - 1	- IODE Issue of Data, Ephemeris - Crs (meters) - Delta n (radians/sec) - M0 (radians)	3X,4D19.12
BROADCAST ORBIT - 2	- Cuc (radians) - e Eccentricity - Cus (radians) - sqrt(A) (sqrt(m))	3X,4D19.12
BROADCAST ORBIT - 3	- Toe Time of Ephemeris (sec of GPS week) - Cic (radians) - OMEGA (radians) - CIS (radians)	3X,4D19.12
BROADCAST ORBIT - 4	- i0 (radians) - Crc (meters) - omega (radians) - OMEGA DOT (radians/sec)	3X,4D19.12
BROADCAST ORBIT - 5	- IDOT (radians/sec) - Codes on L2 channel - GPS Week # (to go with TOE) Continuous number, not mod(1024)! - L2 P data flag	3X,4D19.12
BROADCAST ORBIT - 6	- SV accuracy (meters) - SV health (bits 17-22 w 3 sf 1) - TGD (seconds) - IODC Issue of Data, Clock	3X,4D19.12
BROADCAST ORBIT - 7	- Transmission time of message **) (sec of GPS week, derived e.g. from Z-count in Hand Over Word (HOW)	3X,4D19.12

```
     2.10           OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
teqc  2006Sep26     NGS Web User        20130422 03:41:01UTCPGM / RUN BY / DATE
Solaris 5.9|UltraSparc IIi|cc SC5.5|=+-|*Sparc              COMMENT
     2.11           OBSERVATION DATA    G (GPS)             COMMENT
teqc  2013Mar15     NOAA/NOS/NGS/CORS   20130411 00:19:03UTCCOMMENT
Solaris x86 5.10|AMD64|cc SC5.8 -xarch=amd64|=+|=+          COMMENT
teqc  2013Mar15     NOAA/NOS/NGS/CORS   20130411 00:19:00UTCCOMMENT
BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
 SNR is mapped to RINEX snr flag value [1-9]                COMMENT
  L1: 1 -> 1;  90 -> 5; 210 -> 9                            COMMENT
  L2: 1 -> 1; 150 -> 5; 250 -> 9                            COMMENT
teqc windowed: start @ 2013 Apr 10 00:00:00.000             COMMENT
teqc windowed: delta = 86400.000 sec                        COMMENT
pseudorange smoothing corrections not applied               COMMENT
teqc edited: all GLONASS satellites excluded                COMMENT
ARP8                                                        MARKER NAME
USCG DGPS           U.S. Coast Guard                        OBSERVER / AGENCY
4732137202          TRIMBLE NETRS       1.2-5               REC # / TYPE / VERS
60104294            TRM41249USCG    SCIT                    ANT # / TYPE
  -693632.2200 -5601309.4070  2960670.6680                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     7    L1    L2    C1    P1    P2    S1    S2            # / TYPES OF OBSERV
     5.0000                                                 INTERVAL
teqc windowed: start @ 2013 Apr 10 00:00:00.000             COMMENT
teqc windowed:  end  @ 2013 Apr 10 01:00:00.000             COMMENT
  2013     4    10     0     0    0.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER
 13  4 10  0  0  0.0000000  0 12G 1G 7G 3G 8G 6G28G16G23G13G19G10G11
  -2406934.326 6  -1740672.251 3  24982816.113                    24982832.455
       113.000          93.000
 -22035792.510 9 -16898668.226 7  21276187.682                    21276187.934
       200.000         193.000
 -20667868.082 8 -15499733.991 5  21715106.266                    21715107.446
       173.000         160.000
 -12502584.742 8  -9561502.378 5  23227002.656                    23227005.675
       166.000         140.000
  -6360263.858 7  -4501794.813 4  23759215.171                    23759219.754
       140.000         100.000
  -8240811.526 7  -6118470.153 4  23400191.899                    23400197.146
       160.000         106.000
  15678440.362 6  13268713.332 3  25212997.223                    25213002.163
       106.000          73.000
```

```
#cP2013  4 10  0  0  0.00000000      96 ORBIT IGb08 HLM  IGS
## 1735 259200.00000000   900.00000000 56392 0.0000000000000
+   32   G01G02G03G04G05G06G07G08G09G10G11G12G13G14G15G16G17
+        G18G19G20G21G22G23G24G25G26G27G28G29G30G31G32  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+          0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         2  2  2  2  2  2  2  2  0  3  2  3  2  2  3  2  3
++         2  2  2  2  2  2  2  2  2  0  2  3  3  2  2  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
%c G  cc GPS ccc cccc cccc cccc cccc ccccc ccccc ccccc ccccc
%c cc cc ccc ccc cccc cccc cccc cccc ccccc ccccc ccccc ccccc
%f  1.2500000  1.025000000  0.00000000000  0.000000000000000
%f  0.0000000  0.000000000  0.00000000000  0.000000000000000
%i    0    0    0    0      0      0      0      0         0
%i    0    0    0    0      0      0      0      0         0
/* RAPID ORBIT COMBINATION FROM WEIGHTED AVERAGE OF:
/* cod emr esa gfz jpl ngs sio usn
/* REFERENCED TO IGS TIME (IGST) AND TO WEIGHTED MEAN POLE:
/* PCV:IGS08_1734 OL/AL:FES2004  NONE     Y  ORB:CMB CLK:CMB
*  2013  4 10  0  0  0.00000000
PG01   9009.920621 -19867.672296 -15113.381196     14.794049  8  6  6 124
PG02 -22374.339481  12819.218623  -4895.267320    426.508425 10  5 11 138
PG03  11277.586727 -13441.000766  19370.433516    192.013175  7  4  7 118
PG04 -21927.365932   2439.758373 -15268.098957    185.906865  5  7  9 135
PG05 -18805.028741   5851.477482  17854.558435   -391.067853  8  9  5 118
PG06  14566.719514  -6618.401738  21252.430731    280.190883  7  6  6 106
PG07  -6330.665202 -13685.648321  21884.435990    200.915764  3  7  6 116
PG08 -15510.861869  -6470.215069  20828.601785      5.553553  8  8  7 101
PG09 -22853.919549  11064.462328   7315.364137 999999.999999
PG10 -25143.995001  -8130.418733   4069.345671    -76.151766  8 10 13 143
PG11  10618.701872 -23377.963425  -6651.200669   -370.779680  7  7  3 127
PG12  -1765.537781  15172.934363 -21781.288890    127.520873 10  7  7 115
PG13  -4176.342056 -25527.980601   5599.437184    130.063865  6  7  7 141
PG14  13720.466932  11620.660219 -19404.427202    221.091106  7  9  4 115
PG15  -5276.011714  23092.785698  11724.011344   -114.679936 13  9 12 150
PG16  19468.642991  -3651.225366  17845.717761   -252.568966  8  6  7 105
PG17 -14342.406714 -10816.155437 -19423.372230     31.454082 10 12  9 135
```

```
Antenna Reference Point(ARP): ARANSAS PASS 8 CORS ARP
-----------------------------------------------------
                PID = DK7567


IGS08 POSITION (EPOCH 2005.0)
Computed in Aug 2011 using data through gpswk 1631.
    X =   -693632.869 m     latitude    =  27 50 18.11965 N
    Y =  -5601307.921 m     longitude   = 097 03 33.23857 W
    Z =   2960670.496 m     ellipsoid height =  -16.395   m

IGS08 VELOCITY
Computed in Aug 2011 using data through gpswk 1631.
    VX =  -0.0127 m/yr      northward =  -0.0026 m/yr
    VY =   0.0013 m/yr      eastward  =  -0.0128 m/yr
    VZ =  -0.0028 m/yr      upward    =  -0.0011 m/yr


NAD_83 (2011) POSITION (EPOCH 2010.0)
Transformed from IGS08 (epoch 2005.0) position in Aug 2011.
    X =   -693632.220 m     latitude    =  27 50 18.10343 N
    Y =  -5601309.407 m     longitude   = 097 03 33.20835 W
    Z =   2960670.668 m     ellipsoid height =  -15.081   m

NAD_83 (2011) VELOCITY
Transformed from IGS08 velocity in Aug 2011.
    VX =   0.0004 m/yr      northward =   0.0009 m/yr
    VY =   0.0021 m/yr      eastward  =   0.0001 m/yr
    VZ =  -0.0001 m/yr      upward    =  -0.0019 m/yr
```

HTDP Output

```
****************************************
HTDP (version 3.2.3) OUTPUT

TRANSFORMING POSITIONS FROM ITRF2008 or IGS08        (EPOCH = 01-01-2005 (2005
                         TO ITRF2008 or IGS08        (EPOCH = 04-10-2013 (2013

            INPUT COORDINATES   OUTPUT COORDINATES    INPUT VELOCITY

ARP8
 LATITUDE     27 50 18.11964 N     27 50 18.11877 N       -3.24 mm/yr  north
 LONGITUDE    97 03 33.23856 W     97 03 33.24247 W      -12.94 mm/yr  east
 ELLIP. HT.             -16.395              -16.395 m     0.00 mm/yr  up
 X                  -693632.869          -693632.977 m   -13.03 mm/yr
 Y                 -5601307.921         -5601307.920 m     0.09 mm/yr
 Z                  2960670.496          2960670.472 m    -2.87 mm/yr
```

Some preliminary observations

```
13  4 10  0  0  0.0000000  0 12G 1G 7G 3G 8G 6G28G16G23G13G19G10G11
 -2406934.326 6  -1740672.251 3  24982816.113                    24982832.455
      113.000          93.000
```

Top are SV measured ranges to SV1 (phases, metric ranges and doppler)

```
*  2013  4 10  0  0  0.00000000
PG01   9009.920621 -19867.672296 -15113.381196    14.794049  8  6  6 124
```

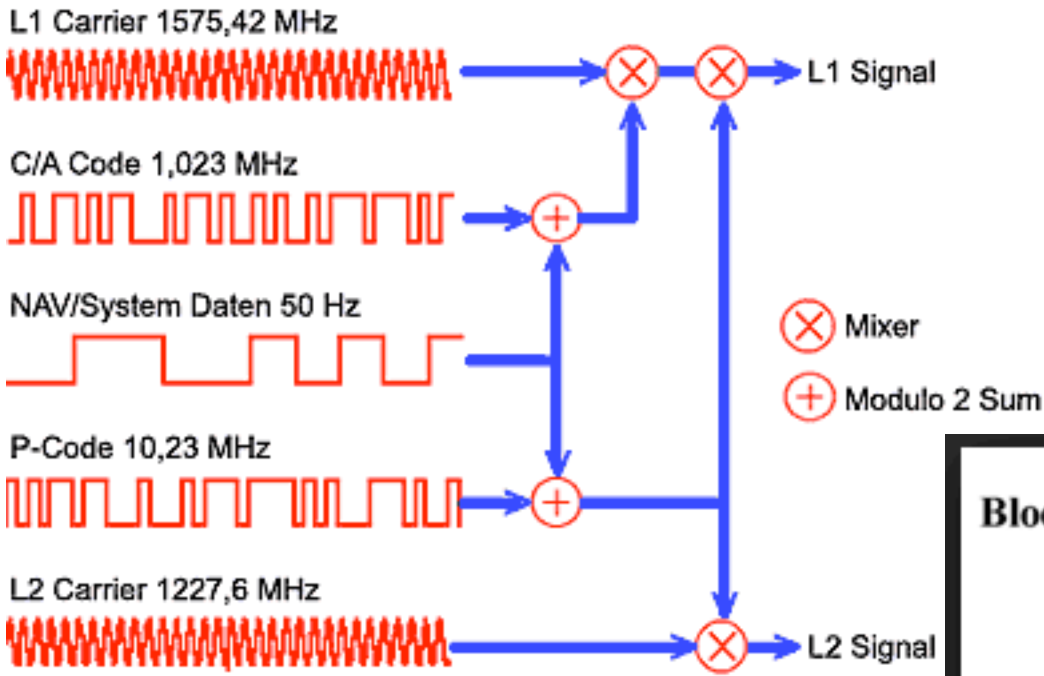
Middle is SV1's position from the precise orbit (in kilometers)

```
 ARP8
  LATITUDE      27 50 18.11964 N      27 50 18.11877 N
  LONGITUDE     97 03 33.23856 W      97 03 33.24247 W
  ELLIP. HT.              -16.395             -16.395 m
  X                   -693632.869         -693632.977 m
  Y                  -5601307.921        -5601307.920 m
  Z                   2960670.496         2960670.472 m
```

Bottom right is position for ARP8 (compatible with observation date/time)

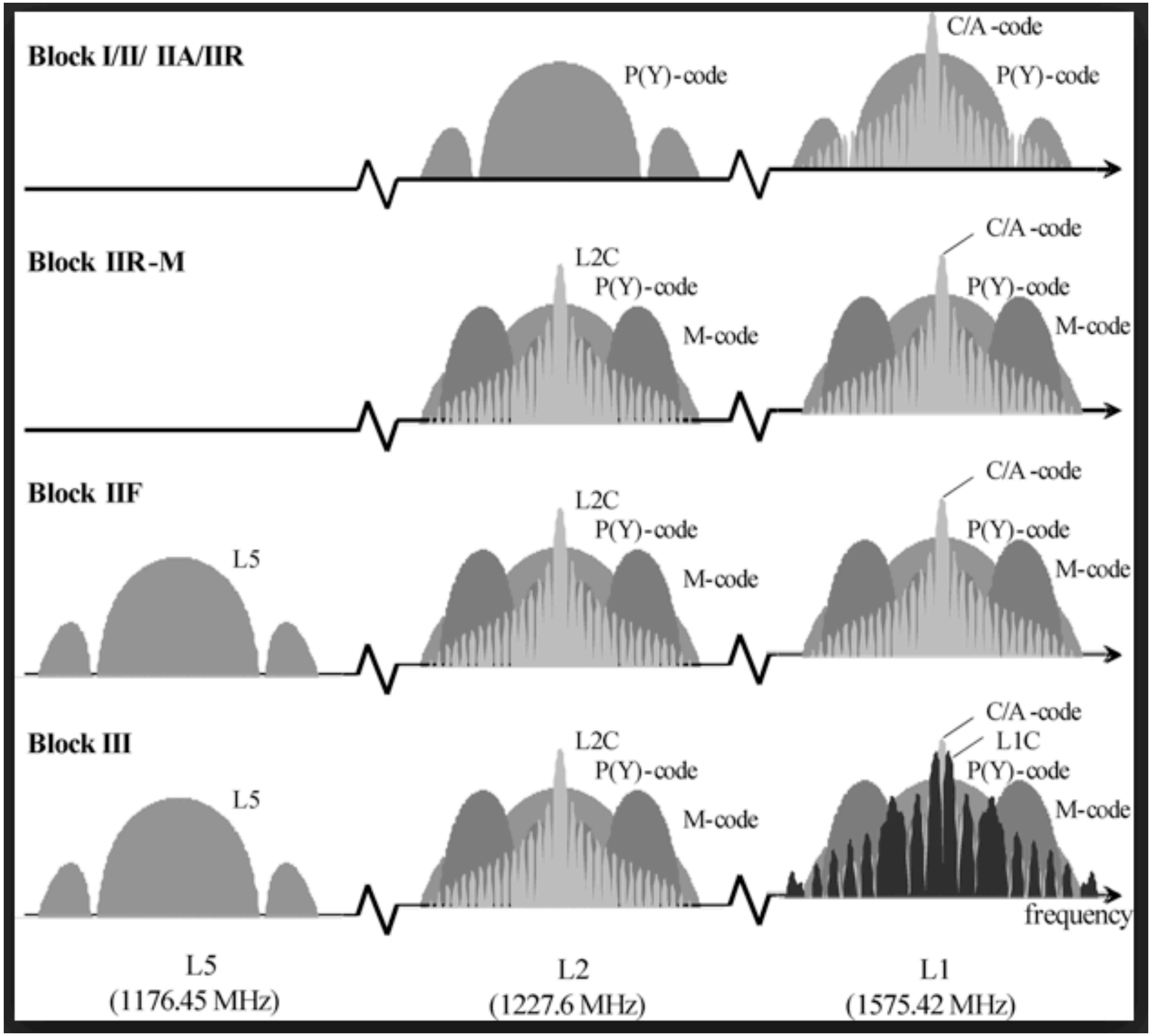
```
How good are raw pseudoranges?

Published ARP8 at time of obs: X =-693632.977m  Y =-5601307.920m        Z =+2960670.472m
IGS Precise Orbit position.    X =+9009920.621m         Y =-19867672.296m       Z =-15113381.196m
Obs file CA1 pseudorange: 24982816.113m P2 pseudorange: 24982832.455m
Differences between L1 and L2 pseudoranges:    -16.342m

Inverse distance using published coordinates ARP8 and SV1: 24987185.689m
Difference between L1 PR and inverse distance:              4369.576m
Difference between L2 PR and inverse distance:              4353.234m
```

GPS Signals



Band frequencies are multiples of the clock's fundamental frequency: 10.23 MHz

L1 = 154	wavelength	19.0 cm
L2 = 120	"	24.4 cm
L5 = 115	"	25.5 cm



Satellite Signals

Current
- L1 – 1575.42 MHz
- L2 – 1227.60 MHz

Signals Modernization

- L2C
  - More reliable measurements
  - Easy to track signal
  - Redundant signal
- L5
  - Improved signal structure
  - Higher transmit power
  - Wider bandwidth
  - Faster signal acquisition

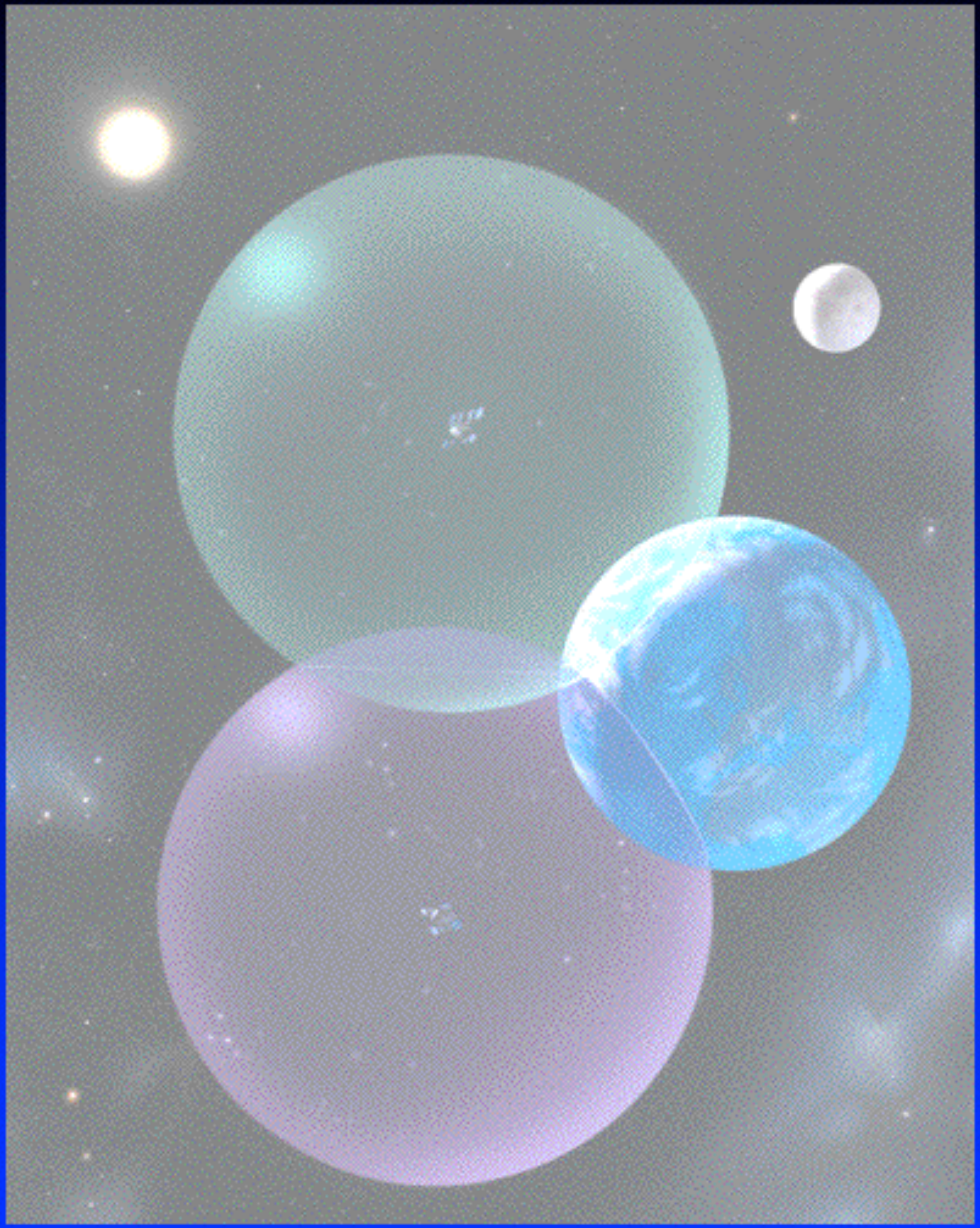
SITECH™

Trimble

Measurements

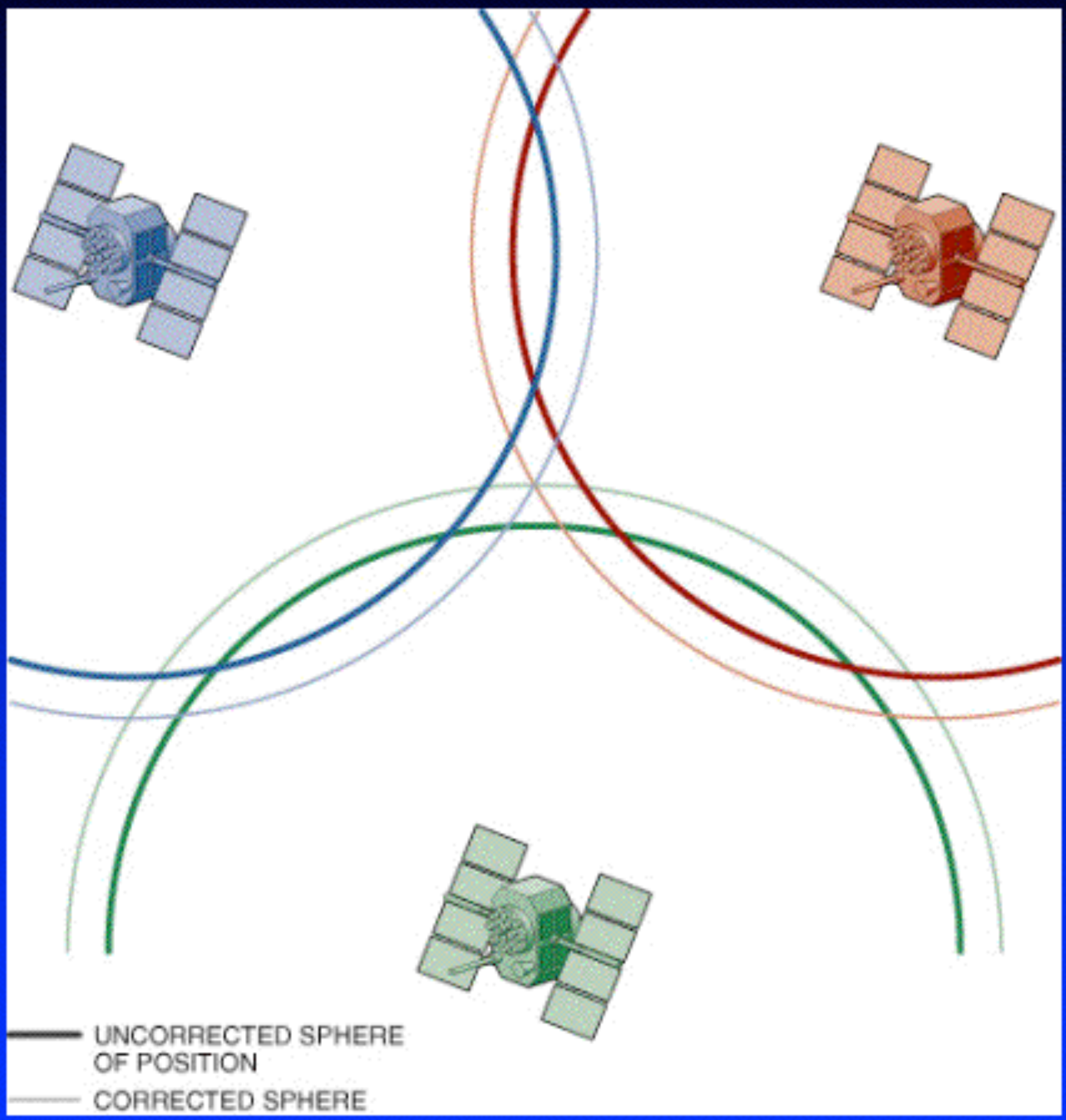
- Measurements:
  - Time difference between signal transmission from satellite and its arrival at ground station (called "pseudo-range", precise to 0.1–10 m)
  - Carrier phase difference between transmitter and receiver (precise to a few millimeters) but initial values unknown (ie., measures change in range to satellites).
  - In some case, the integer values of the initial phase ambiguities can be determined (bias fixing)
- All measurements relative to "clocks" in ground receiver and satellites (potentially poses problems).

Position Determination (perfect clocks).



- Three satellites are needed for 3-D position with perfect clocks.
- Two satellites are OK if height is known)

11/19/01



Position determination: with clock errors: 2-D case

- Receiver clock is fast in this case, so all pseudo-ranges are short

Positioning

- For pseudo-range to be used for positioning need:
  - Knowledge of errors in satellite clocks
  - Knowledge of positions of satellites
- This information is transmitted by satellite in “broadcast ephemeris”
- “Differential” positioning (DGPS) eliminates need for accurate satellite clock knowledge.

- **BIASES**
  - Satellite dependent:
    - Ephemeris uncertainties
    - Sate llite clock uncertainties
    - Selective Availability effects
  - Receiver dependent:
    - Receiver clock uncertainties
    - Reference station coordinate uncertainties
  - Receiver-Satellite (or Observation) dependent:
    - Ionospheric delay
    - Tropospheric delay
    - Carrier phase ambiguity
- **ERRORS**
    - Unmodelled, residual biases
    - Carrier phase cycle slips
    - Multipath disturbance
    - Antenna phase centre offset
    - Random observation error (noise)

Effects of Biases and Errors in GPS can be modeled or impact mitigated with additional information e.g. precise orbits, use of antenna phase models, data combinations, etc.

Clock effects

- GPS is controlled by 10.23 MHz oscillators
- On the Earth's surface these oscillators are set to $10.23x(1\text{-}4.4647x10^{-10})$ MHz (39,000 ns/day rate difference)
- This offset accounts for the change in potential and average velocity once the satellite is launched.
- The first GPS satellites had a switch to turn this effect on. They were launched with "Newtonian" clocks

Relativistic effects

- General relativity affects GPS in three ways
  - Equations of motions of satellite
  - Rates at which clock run
  - Signal propagation
- In our GPS analysis we account for the second two items
- Orbits only integrated for 1-3 days and equation of motion term is considered small

Phase measurements

$$\phi_k^p(t_r) = \phi_k(t_r) - \phi_r^p(t_r) + N_k^p(1)$$

- The carrier phase is the difference between phase of receiver oscillator and signal received plus the number of cycles at the initial start of tracking
- The received phase is related to the transmitted phase and propagation time by

$$\phi_r^p(t_r) = \phi_t^p(t_t) = \phi_t^p(t_r - \rho_k^p / c) = \phi_t^p(t_r) - \dot{\phi}^p(t_r) \cdot \rho_k^p / c$$

Precision of phase measurements

- Nominally phase can be measured to 1% of wavelength (~2mm L1 and ~2.4 mm L2)
- Again effected by multipath, ionospheric delays (~30m), atmospheric delays (3-30m).
- Since phase is more precise than range, more effects need to be carefully accounted for with phase.
- Precise and consistent definition of time of events is one the most critical areas
- In general, phase can be treated like range measurement with unknown offset due to cycles and offsets of oscillator phases.