Lecture 13 – “Geodetic Reference Systems”

GISC-3325

27 February 2012

Updates

- Read Chapter 8 for this Wednesday and next Monday.
- **Be sure to select your article for the oral (and written report)**. Must be from Journal of Geodesy or GPS Solutions (both available through campus computers).
- Good article on datum transformations at: http://mycoordinates.org/choosing-the-best-path-global-to-national-coordinate-transformations/

3D Coordinate Systems

- Geodetic (Curvilinear) Coordinates
  - Latitude, longitude and ellipsoid height
  - Right-handed, earth-centered earth-fixed, positive east
- Geocentric (Cartesian) Coordinates
  - X, Y, Z
  - Likewise, ECEF, right-handed,
  - Orthogonal

GPS vectors

Difference in geocentric coordinates.

C00011001	-387714127	7	-289669536	23	-652818377	14	R0727ATXCCR0727AD387
C00011002	-376027966	7	-211837717	21	-499460662	13	R0727ATXCCR0727AZ380
C00011003	7427420	5	-230775488	21	-437730506	12	R0727ATXCCR0727A6604
C00011004	106489542	5	-146932779	19	-252785019	11	R0727ATXCCR0727A6139
C00011006	193199270	7	-79886176	23	-104306881	12	R0727ATXCCR0727A5792
C00011005	204114913	7	-105324866	24	-149901430	12	R0727ATXCCR0727A5870
C00010002	380529109	6	2449912	14	95609032	7	R0727ATXCCA0727AARP5
[illegible]	[illegible]	5	[illegible]	20	[illegible]	10	[illegible]

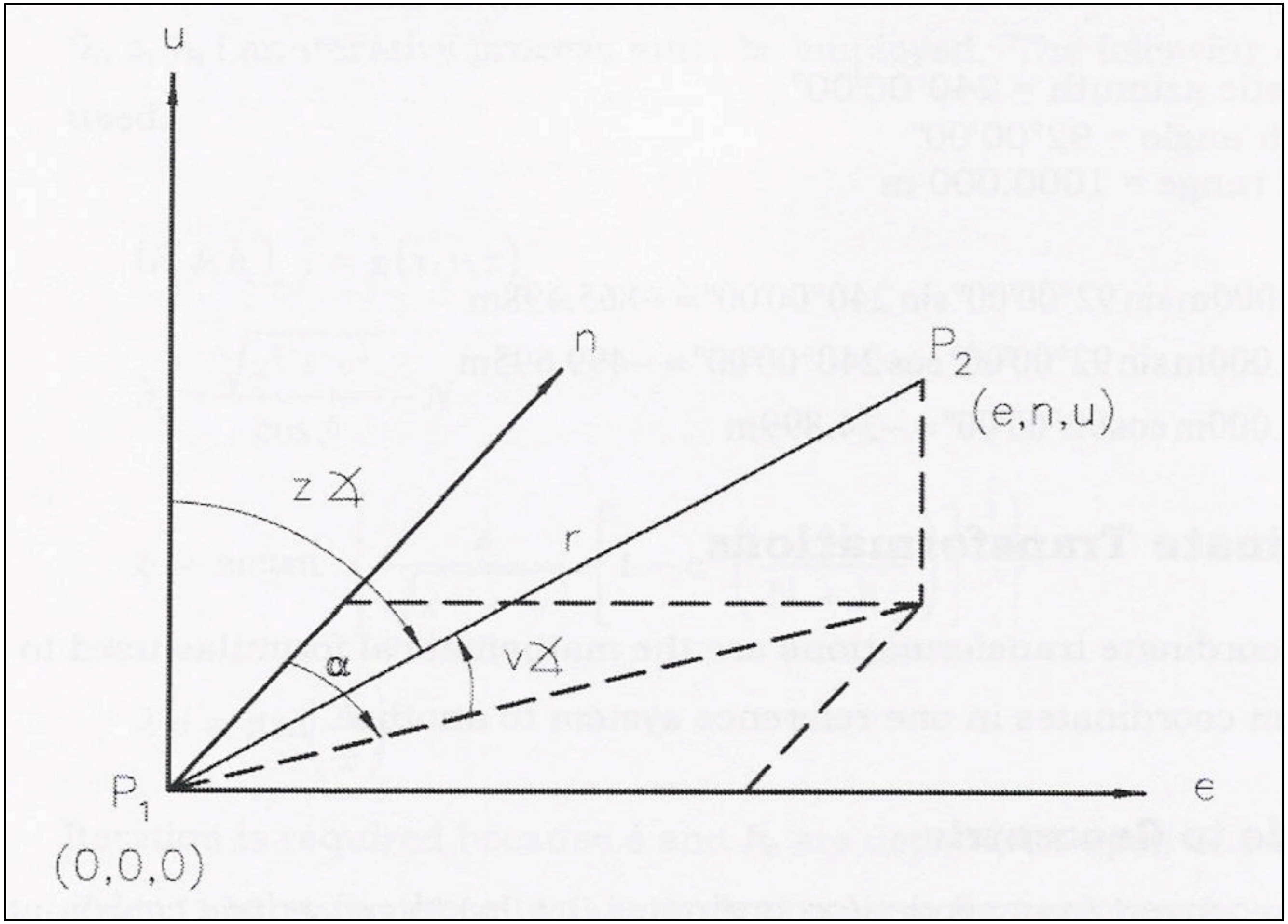
Both difference in geocentric coordinates and changes in local geodetic horizon coordinates.

295	DX	38052.9122	38052.9139	-0.0017	0.0003	-6.0425	0.0007	(1) TXCC
296	DY	244.9962	245.0011	-0.0049	0.0010	-4.7281	0.0015	(2) ARP5
297	DZ	9560.9018	9560.8997	0.0021	0.0006	3.7777	0.0008	25.48 2987A
	DN	10810.5353	10810.5358	-0.0005	0.0003			
	DE	37717.7317	37717.7328	-0.0010	0.0002		Vector 13 Solution	13
	DL			0.0011	0.0004		Project ID =	
	DU	-6.0264	-6.0319	0.0055	0.0012			

Local Geodetic Horizon (LGH)

- ECEF, right-handed, orthogonal, 3-D
- **Origin at any point specified**
  - N in meridian plane oriented toward N pole
  - U normal to ellipsoid at origin
  - E perpendicular to meridian plane
- Depending on software (or algorithm) values can appear as ENU or NEU.

LGH



$$e = r\cos(v\angle)\sin\alpha = r\sin(z\angle)\sin\alpha$$

$$n = r\cos(v\angle)\cos\alpha = r\sin(z\angle)\cos\alpha$$

$$u = r\sin(v\angle) = r\cos(z\angle)$$

$$\alpha = \arctan\left(\frac{e}{n}\right)$$

Geodetic azimuth.

$$r = \left(e^2 + n^2 + u^2\right)^{1/2}$$

Text

Mark-to-mark slant range.

$$v\angle = \arcsin\left(\frac{u}{r}\right)$$

Vertical or zenith angle.

$$z\angle = \arccos\left(\frac{u}{r}\right)$$

Either vertical angle or zenith angles can be used.

Geodetic to Geocentric Coordinate Conversions

The conversion from curvilinear geodetic (λ, ϕ, h) to Cartesian (x, y, z) coordinates is given by the well-known equations:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ [N(1-e^2)+h]\sin\phi \end{Bmatrix} \qquad (3)$$

$$N = \frac{a\cos\phi}{\cos\phi\left(1-e^2\sin^2\phi\right)^{1/2}},$$

$$\therefore \quad N = \frac{a}{\left(1-e^2\sin^2\phi\right)^{1/2}}.$$

Geocentric to Geodetic

$$(\lambda, \phi, h_e)_{a,f} = g(x, y, z)$$

$$h_e = \frac{\sqrt{x^2 + y^2}}{\cos\phi} - N$$

$$\phi = \arctan \left\{ \frac{z}{\sqrt{x^2 + y^2}} \left[1 - e^2 \left(\frac{N}{N + h_e} \right) \right]^{-1} \right\}$$

$$\lambda = \arctan\left(\frac{y}{x}\right)$$

$$N = \frac{a}{\left(1 - e^2 \sin^2\phi\right)^{1/2}}.$$

Geocentric to Geodetic

- We use ellipsoid parameters (a, f^{-1})
- Calculate preliminary values (set: $h = 0$)
  - $Lat_1 = atan((Z / sqrt(x^2+y^2))*(1/(1-e^2))$
  - $N_1 = a / sqrt(1-e^2*sin(Lat_1)^2)$
  - $h_1 = (sqrt(x^2+y^2)/cos(Lat_1))-N_1$
- We iterate using these starting values
- We stop iterating when the shift in ellipsoid height is within our accuracy goal.

2D-Coordinate Transformations

- Given
  - $x = r * \cos(\gamma)$
  - $y = r * \sin(\gamma)$
- Rotate coordinate system by Θ
  - $x' = r * \cos(\gamma - \Theta)$
  - $y' = r * \sin(\gamma - \Theta)$
- Use following trig identities to solve:
  - $\cos(\gamma - \Theta) = \cos \gamma \cos \Theta + \sin \Theta \sin \gamma$
  - $\sin(\gamma - \Theta) = \sin \gamma \cos \Theta - \cos \gamma \sin \Theta$

Translation

- If we shift the origin we can update coordinates by merely adding/subtracting shift from matching coordinate.
  - $x' = x - tx$
  - $y' = y - ty$

Scale change

- We can scale coordinates to account for issues like m to ft.
  - $x' = s * x$
  - $y' = s * y$

Four-parameter transformation

- Combines rotations, translations and scale in one operation. Two-dimension case.
  - $x' = s*(x*\cos\Theta + y*\sin\Theta) + tx$
  - $y' = s*(-x*\sin\Theta + y*\cos\Theta) + ty$
- Matrix form is simpler

Three-Dimensional Transformation

- 7-parameters
  - one scale
  - three rotations along X,Y,Z axes
  - three translations in X,Y,Z

Euler matrices and 7-parameter

$$D \equiv \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C \equiv \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$B \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

Matrix D for rotation on Z axis

Matrix C for rotation of Y axis

Matrix B for rotation of X axis

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + (1 + \Delta L)\begin{pmatrix} 1 & \omega_3 & -\omega_2 \\ -\omega_3 & 1 & \omega_1 \\ \omega_2 & -\omega_1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_D$$

Euler matrices

In $\mathbb{R}^3$, coordinate system rotations of the *x*-, *y*-, and *z*-axes in a counterclockwise direction when looking towards the origin give the matrices

$$\mathrm{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{4}$$

$$\mathrm{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \tag{5}$$

$$\mathrm{R}_z(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

7-parameters to transform NAD83 to ITRF96

$$T_X(t) = 0.9910\, m \tag{5}$$

$$T_Y(t) = -1.9072\, m \tag{6}$$

$$T_Z(t) = -0.5129\, m \tag{7}$$

$$R_X(t) = [125033 + 258(t - 1997.0)]\,(10^{-12})\, radians \tag{8}$$

$$R_Y(t) = [46785 - 3599(t - 1997.0)]\,(10^{-12})\, radians \tag{9}$$

$$R_Z(t) = [56529 - 153(t - 1997.0)]\,(10^{-12})\, radians \tag{10}$$

$$S(t) = 0.0\, (unitless) \tag{11}$$

When transforming from NAD 83 to ITRF96,

Table 1. Transformation Parameters between Different Frames for $t_0 = 1997.00$

Parameter	Units	ITRF00→ITRF97	ITRF97→ITRF96	ITRF96→NAD 83
$T_x(t_0)$	meters	+0.0067	−0.00207	+0.9910
$\dot{T}_x$	meters/year	+0.0000	+0.00069	+0.0[a]
$T_y(t_0)$	meters	+0.0061	−0.00021	−1.9072
$\dot{T}_y$	meters/year	−0.0006	−0.00010	+0.0[a]
$T_z(t_0)$	meters	−0.0185	+0.00995	−0.5129
$\dot{T}_z$	meters/year	−0.0014	+0.00186	+0.0[a]
$\varepsilon_x(t_0)$	mas	+0.0[a]	+0.12467	+25.79
$\dot{\varepsilon}_x$	mas/year	+0.0[a]	+0.01347	+0.0532
$\varepsilon_y(t_0)$	mas	+0.0[a]	−0.22355	+9.65
$\dot{\varepsilon}_y$	mas/year	+0.0[a]	−0.01514	−0.7423
$\varepsilon_z(t_0)$	mas	+0.0[a]	−0.06065	+11.66
$\dot{\varepsilon}_z$	mas/year	−0.02	+0.00027	−0.0316
$s(t_0)$	ppb	+1.55	−0.93496	+0.0[a]
$\dot{s}$	ppb/year	+0.01	−0.19201	+0.0[a]

Note: mas≡milliarc second. Counterclockwise rotations of axes are assumed positive; 1 ppb$=10^{-3}$ ppm.

[a]Values set to zero by definition.

Table 2. Parameters Adopted for Transformation ITRF00→NAD 83 (CORS96)

Parameter epoch: $t_0 = 1997.00$	Definition	Units	Values at t_0
$T_x(t_0)$	x-shift	meters	+0.9956
$T_y(t_0)$	y-shift	meters	−1.9013
$T_z(t_0)$	z-shift	meters	−0.5215
$\varepsilon_x(t_0)$	x-rotation	mas	+25.915
$\varepsilon_y(t_0)$	y-rotation	mas	+9.426
$\varepsilon_z(t_0)$	z-rotation	mas	+11.599
$s(t_0)$	scale	ppb	+0.62
$\dot{T}_x$	x-shift rate	meters/year	+0.0007
$\dot{T}_y$	y-shift rate	meters/year	−0.0007
$\dot{T}_z$	z-shift rate	meters/year	+0.0005
$\dot{\varepsilon}_x$	x-rotation rate	mas/year	+0.067
$\dot{\varepsilon}_y$	y-rotation rate	mas/year	−0.757
$\dot{\varepsilon}_z$	z-rotation rate	mas/year	−0.051
$\dot{s}$	scale rate	ppb/year	−0.18

Note: mas≡milliarc second. Counterclockwise rotation of axes are assumed positive; 1 ppb$=10^{-3}$ ppm.