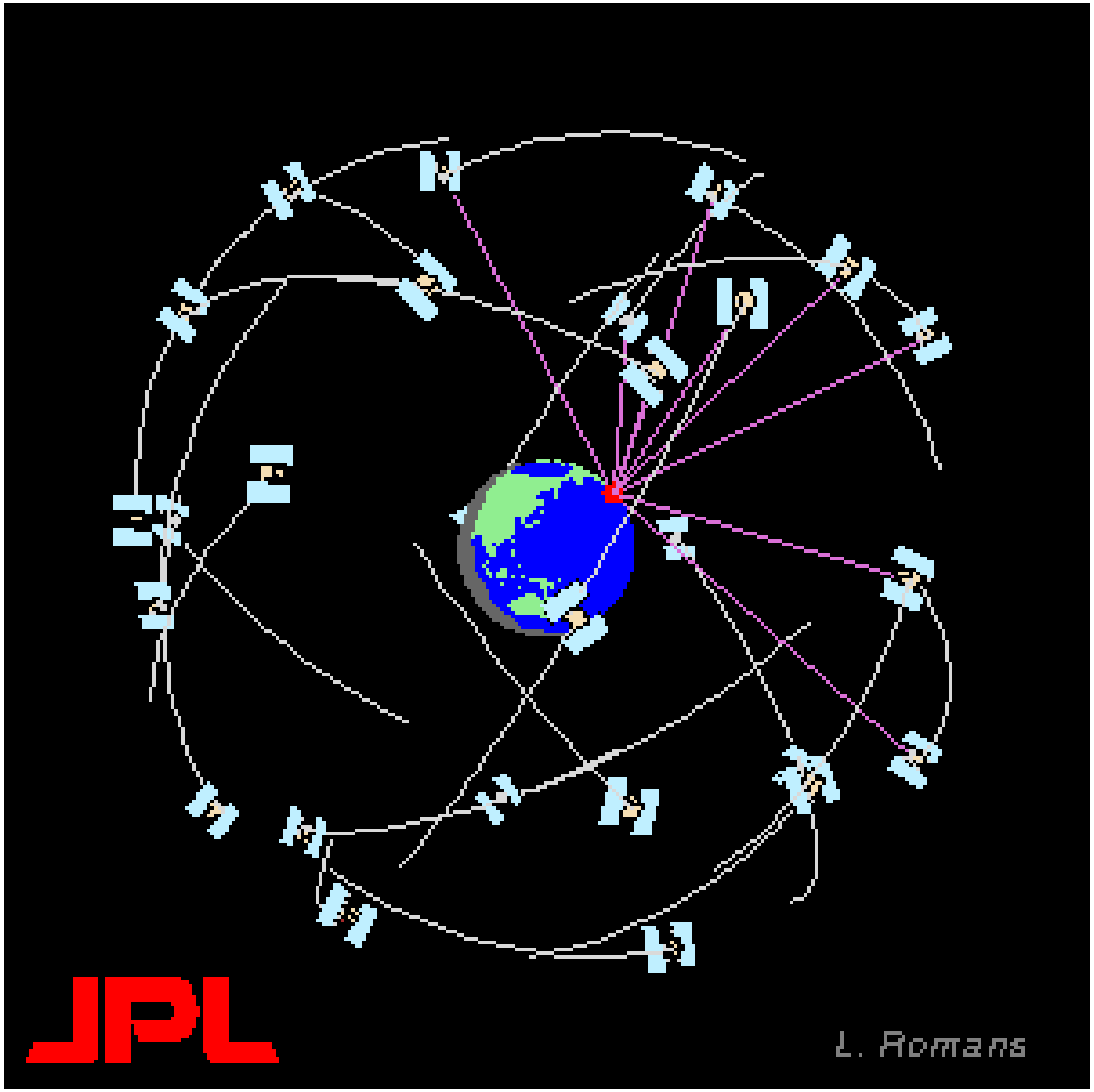
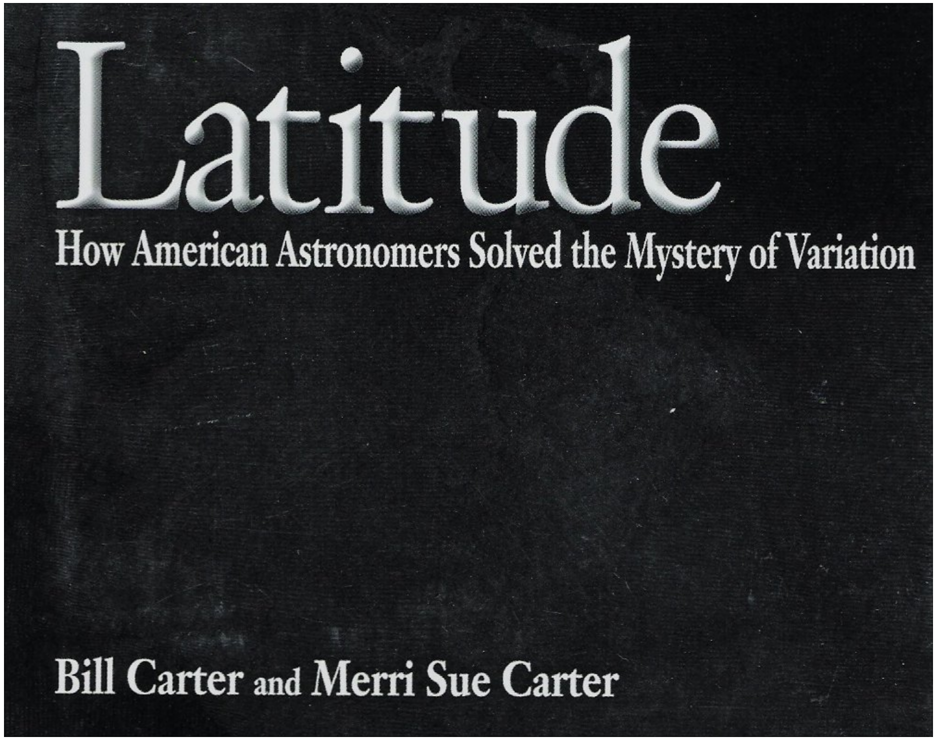

JPL
L. Romans

Lecture 4 – Spherical Trigonometry and related topics

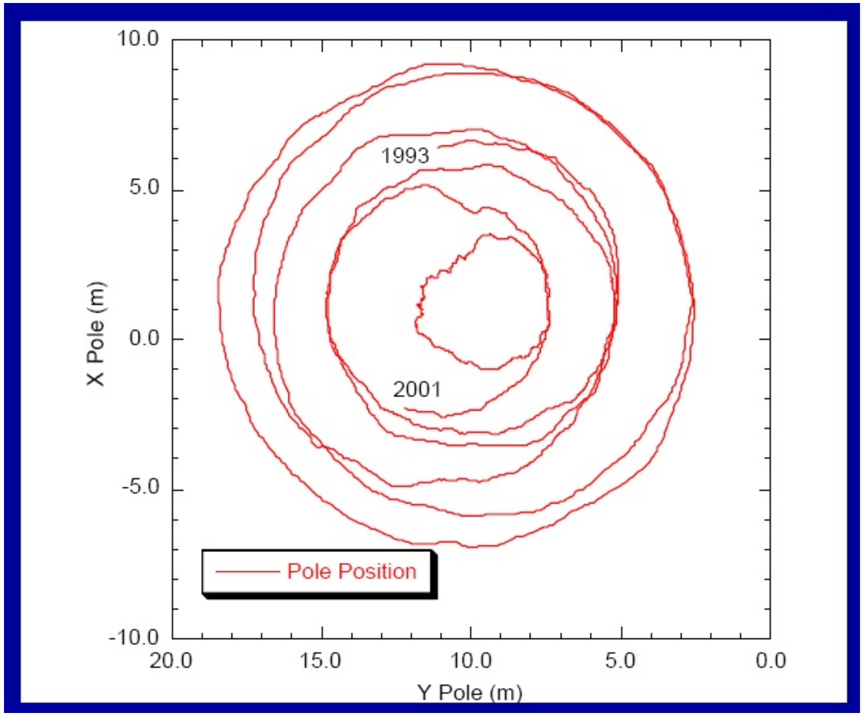
GISC-3325

24 January 2007

Another book recommendation



By Bill Carter and Merri Sue Carter, Naval Institute Press, Annapolis, Maryland 2002



Review

- Latitude and Longitude can uniquely and meaningfully describe where we are on the earth.
- We can also express positions *on a sphere* using 3-D Cartesian coordinates [X;Y;Z] using simple geometric relationships.

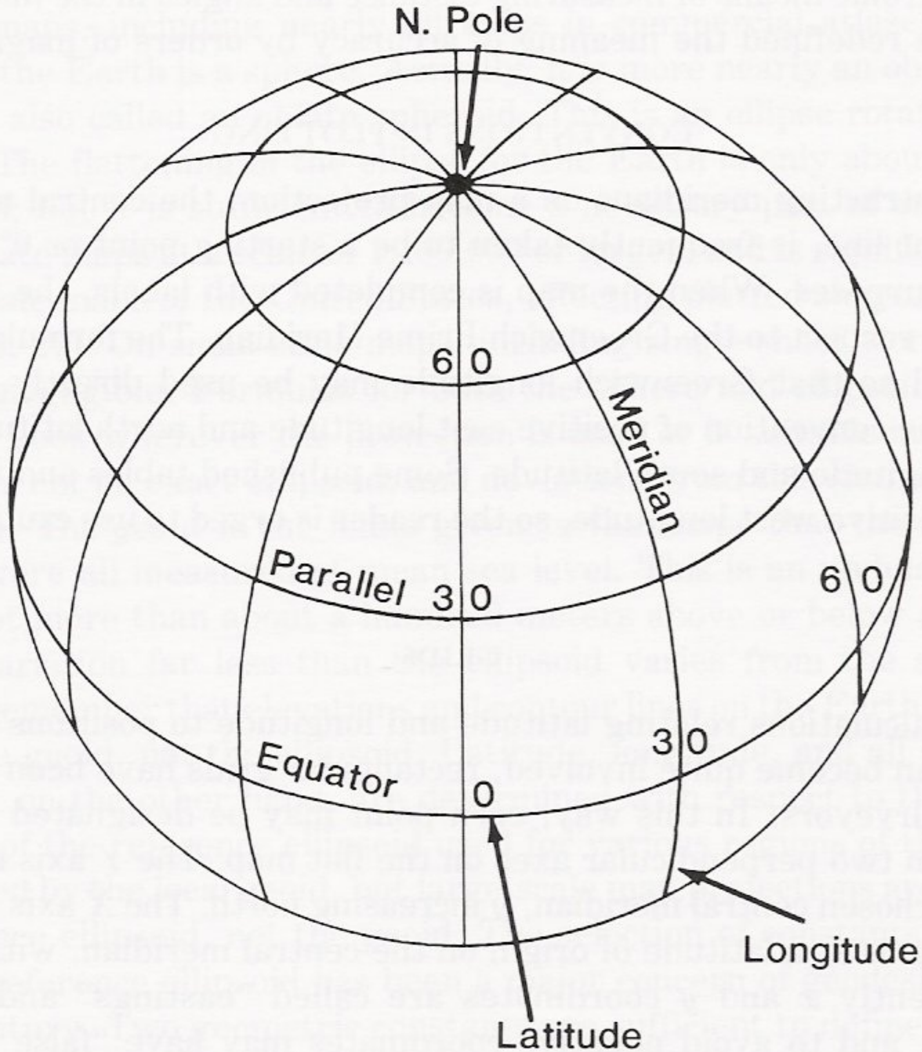
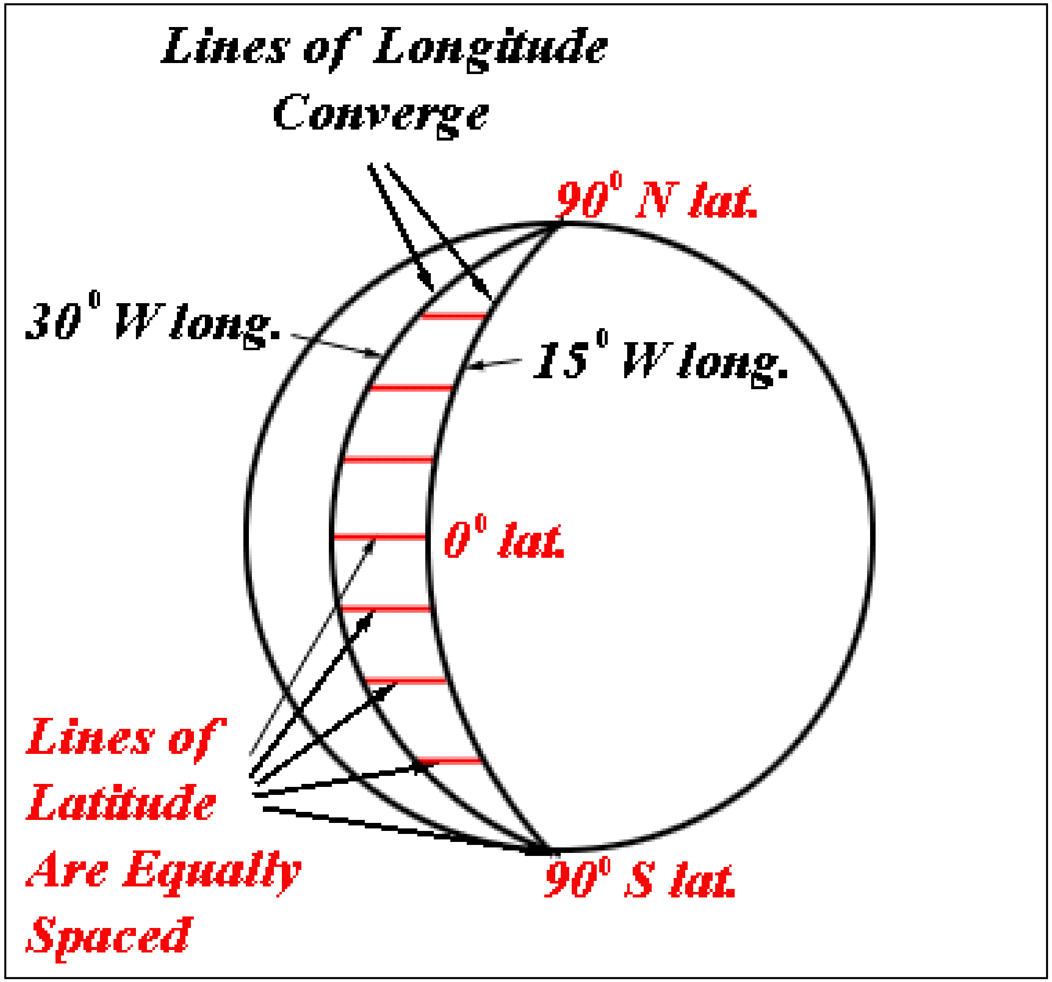


FIGURE 2.—Meridians and parallels on the sphere.


Lines of Longitude
Converge
90⁰ N lat.
30⁰ W long.
15⁰ W long.
0⁰ lat.
Lines of
Latitude
Are Equally
Spaced
90⁰ S lat.

Textbook error

- See page 23.
- Author misplaces the decimal point where he converts DMS to Decimal
  - 31.315278 should be 3.1315278
- The answer to problem 2.2 is correct.

Homework Answer

- Problem: Use the position of station BLUCHER to determine the distance on a spherical earth (with radius 6,378,000 m) from the equator.
  - BLUCHER: 27-42-52.08857N

Editor - C:\MATLAB71\work\homework1_v2.m

```
% Homework 1
blucher = (27+(42/60)+(52.08857/3600))*(pi/180)
r = 6378000;
dist_B_2_equator = r*blucher;
sprintf('\n\tDistance from the equator to BLUCHER is: %10.2f', dist_B_2_equator)
BL_dec_deg = (27+(42/60)+(52.08857/3600));
a = (BL_dec_deg * r*2*pi)/360;
sprintf('\n\tDistance via method of Eratosthenes: %10.2f', a)
```

Both methods yield: 3,085,094 m

What about using INVERSE?

Output from INVERSE

```
Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a   =    6378137.0000
Polar axis,         b   =    6356752.3141
Inverse flattening, 1/f =  298.25722210088

First  Station : equator
 ----------------
  LAT =   0  0  0.00000 North
  LON =  97 19 44.31265 West

Second Station : BLUCHER
 ----------------
  LAT =  27 42 52.08857 North
  LON =  97 19 44.31265 West

Forward azimuth        FAZ =   0  0  0.0000 From North
Back azimuth           BAZ = 180  0  0.0000 From North
Ellipsoidal distance     S =   3066800.0198 m
```

NGS HOME PAGE

3,066,800 meters

10,061,660 feet (US Survey)

1,905.6 miles (statute)

Our result using a radius of 6,378,000 meters is 3,085,094 m

A difference of 18,294 m!

Why??

UNITS ARE IMPORTANT

US v INTL feet

STATUTE v NAUTICAL MILES

Feet are Feet?

- Conversions from meters to feet (and inverse) are complicated by two units of feet.
  - U.S. Survey foot = 0.30480061… meters
    - 1200/3937 meters (exactly)
  - International foot = 0.3048 meters (exactly)
    - 2.54 cm = 1 inch

DMS <-> Radian

- To convert degrees to radians
  - Convert DD MM SS.sssss to decimal
    - Deg + min/60 + sec/3600
  - Convert decimal degrees to radians
    - Multiply by pi/180
- To convert radians to decimal
  - decDeg = Radian value * 180/pi
  - Deg = floor(decDeg)
  - Min = floor((decDeg-Deg)*60)
  - Sec = decDeg*3600-(Deg*60)-(Min*3600)

The meter

- There were great difficulties in commerce due to varying length (and other) units.
- The French Academy of Science was charged with standardizing the measurement unit.
- Original proposal was to use the period of a pendulum.
- Instead, in 1790 the Academy recommended that a meter unit be based on one-millionth of the distance from the Equator to the North Pole.

How well did they do?

Output from INVERSE

```
Ellipsoid : Clarke 1866     (NAD27)
Equatorial axis,     a    =    6378206.4000
Polar axis,          b    =    6356583.8000
Inverse flattening, 1/f =  294.97869821380

First  Station :
 ----------------
  LAT =   0   0   0.00000 North
  LON = 100   0   0.00000 West

Second Station :
 ----------------
  LAT =  90   0   0.00000 North
  LON = 100   0   0.00000 West

Forward azimuth          FAZ =   0   0   0.0000 From North
Back azimuth             BAZ = 180   0   0.0000 From North
Ellipsoidal distance       S =   10001888.0430 m
```

NGS HOME PAGE

Output from INVERSE

```
Ellipsoid : GRS80 / WGS84   (NAD83)
Equatorial axis,     a    =    6378137.0000
Polar axis,          b    =    6356752.3141
Inverse flattening, 1/f =  298.25722210088

First  Station :
 ----------------
  LAT =   0   0   0.00000 North
  LON = 100   0   0.00000 West

Second Station :
 ----------------
  LAT =  90   0   0.00000 North
  LON = 100   0   0.00000 West

Forward azimuth          FAZ =   0   0   0.0000 From North
Back azimuth             BAZ = 180   0   0.0000 From North
Ellipsoidal distance       S =   10001965.7292 m
```

NGS HOME PAGE

Evolution of the meter

- The original measurement was in error due to unknown magnitude of Earth's flattening.
  - The unit was transferred to a platinum-iridium alloy bar kept in Paris (1874)
  - The unit was updated in (1889) to a bar composed of 90% platinum
- In 1960 a new definition was adopted that was based on krypton-86 radiation wavelength.
- Meter is the length of the path traveled by light *in a vacuum* during the time interval of:
  - 299 792 458 s-1 (299 792 458 meters per sec)

Distance on a sphere

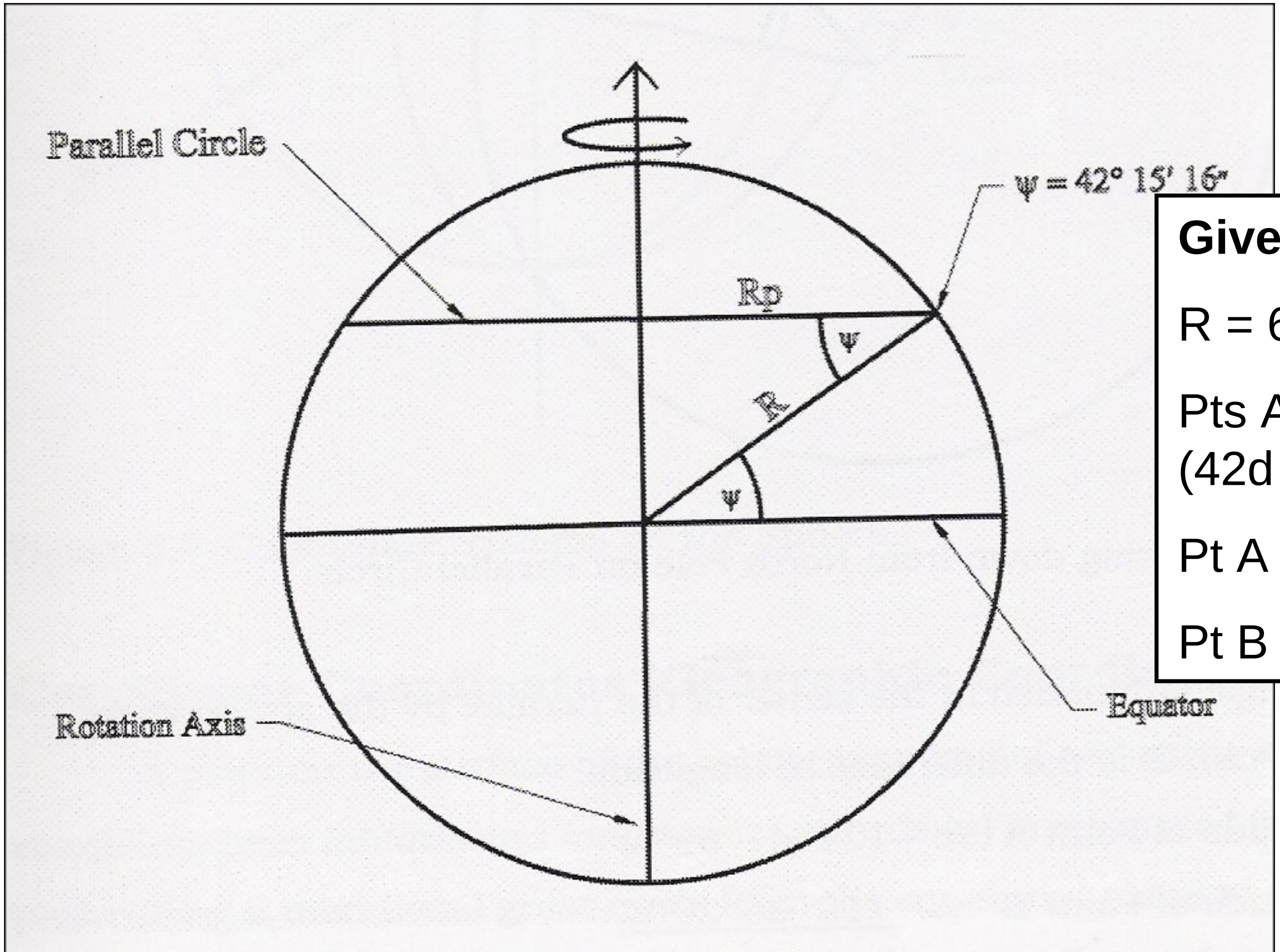


Figure 3.2 Radius of Parallel Circle.

Given

R = 6,371,000 m

Pts A and B on same Latitude (42d 15m 16s N)

Pt A Long: (121d 47m 09s W

Pt B Long: (119d 36m 29s W)

Compute radius of parallel circle by solving right triangle.

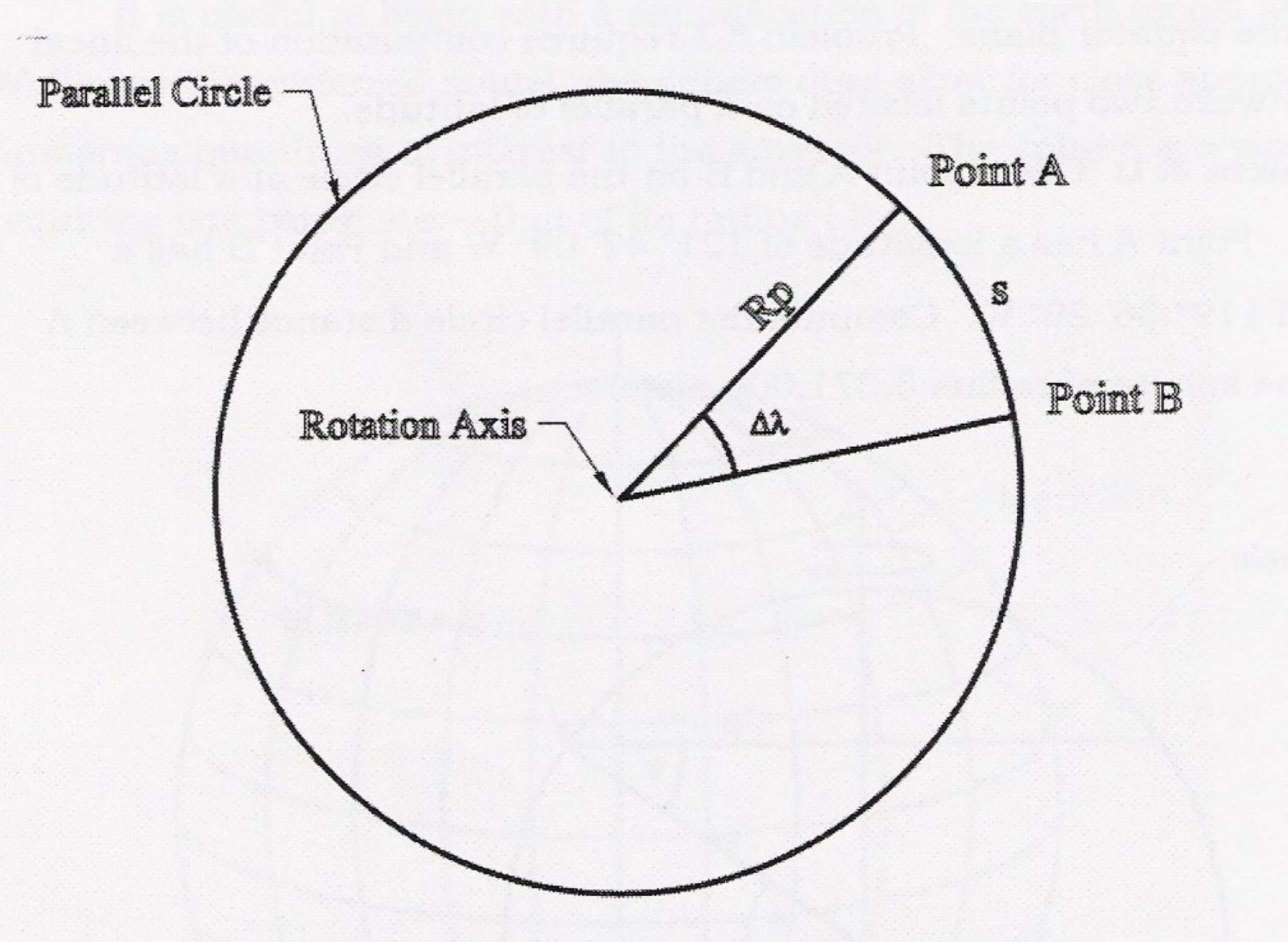


Figure 3.3 Looking down from North Pole on Parallel Circle.

Subtract longitudes to get angle. s = Rp * angle (in radians)

For this problem s = 179,237 meters

Spherical Triangles

- Used in great circle navigation.
- Sides and angle are measured using arc measures
- Located on the surface of the sphere with sides formed by great circle arcs.
  - N.B. great circles are planes through the center of the Earth
- The shortest distance between points.
  - *Not exactly*

Spherical trigonometry

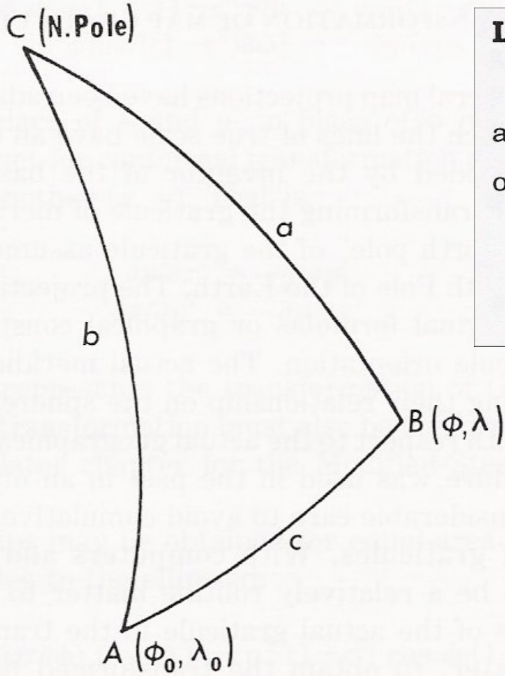


FIGURE 5.—Spherical triangle.

$$\cos c = \cos b \cos a + \sin b \sin a \cos C$$

Law of Cosines

There are two forms for the law of cosines - one for sides and one for angles. Note that the forms are both *cyclic*, i.e., a family of formulas may be obtained by cycling the side and angle variables.

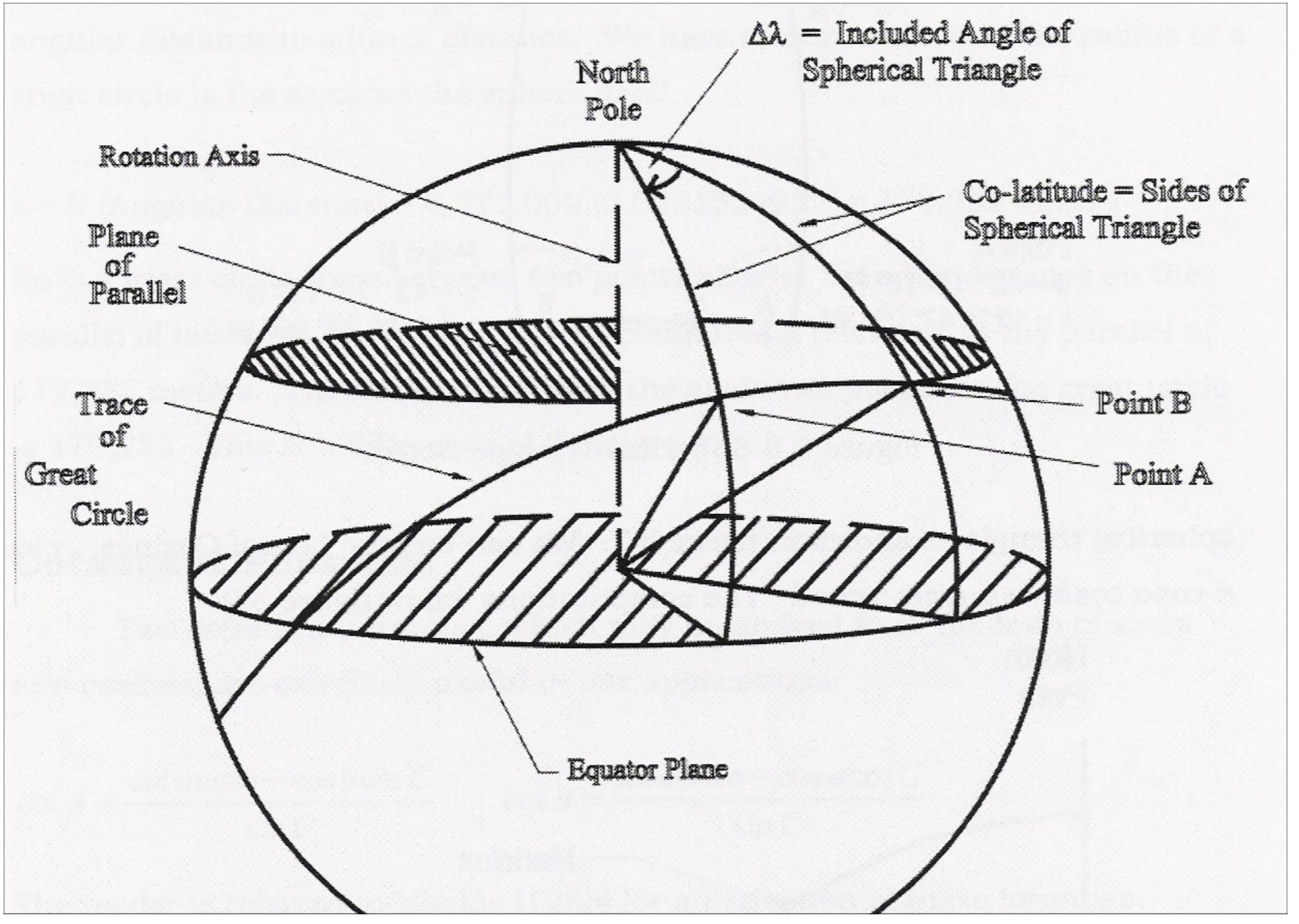
$\cos a = \cos b \cos c + \sin b \sin c \cos A$ (law of cosines for sides).

$\cos A = -\cos B \cos C + \sin B \sin C \cos a$ (law of cosines for angles)

More damned definitions

- **Normal section** is a plane that contains the normal to the sphere at the occupied point and another point of interest.
- **Horizontal angle** is measured between two normal sections with respect to the instrument location.
- **Azimuth** is measured from the normal section containing the N pole clockwise to the normal section containing the other point
- **All normal sections on the sphere intersect the sphere along great circle arcs**.

Spherical Triangle



Problem in Text

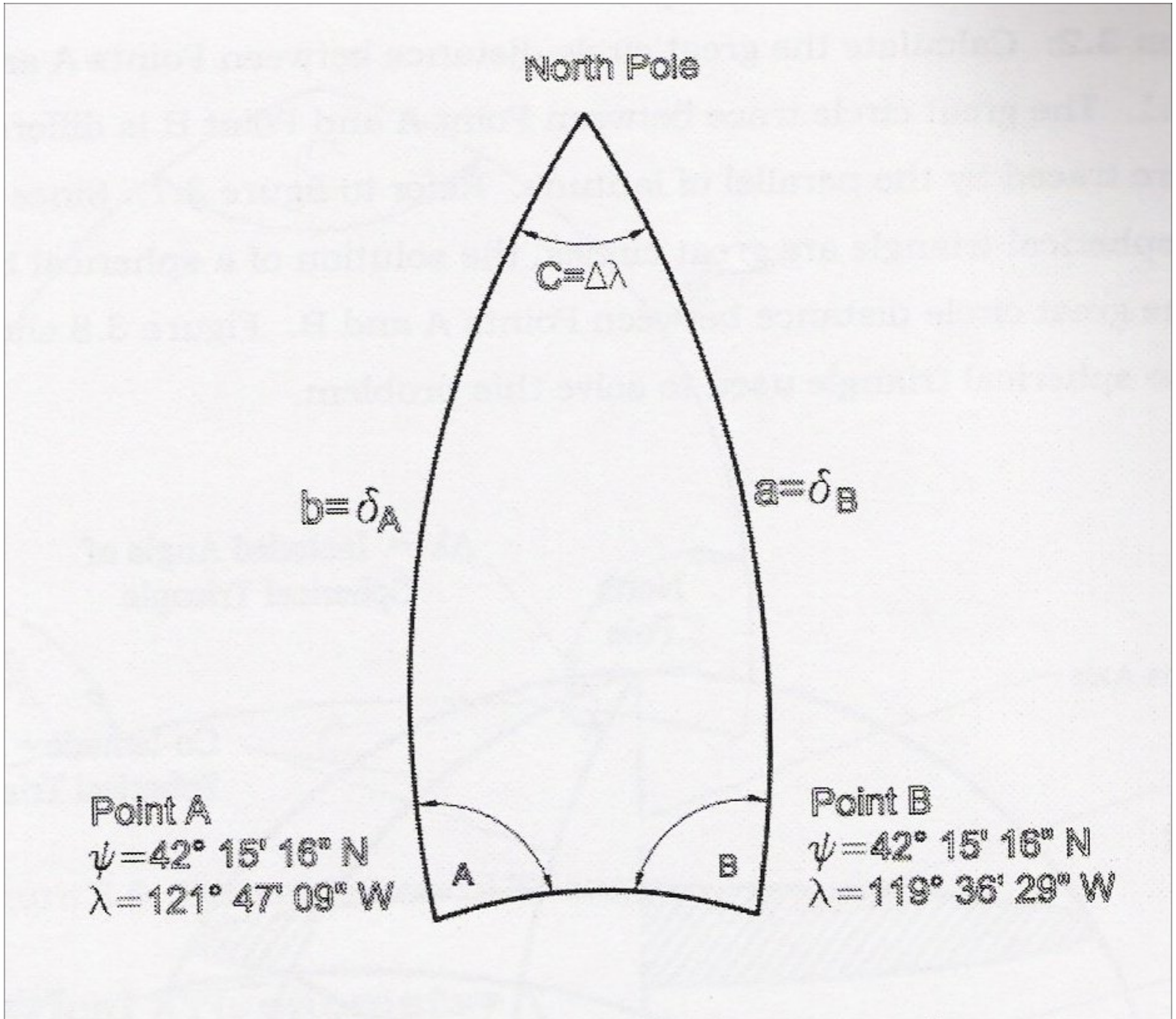


Figure 3.8 Spherical Triangle Detail.

Distance Calculations

- Determine the co-latitudes (90d – latitude) at points A and B.
  - These are the lengths C to B (side a) and C to A (side b).
- Compute the difference in Longitudes
  - This is the angle at C
- As Law of Sines is ambiguous for angles in excess of 90d, we use Law of Cosines to solve for distance side c
  - Cos(C) = cos(A)cos(B)+sin(A)sin(B)cos(C) .
- Distance = r * C (N.B. radius of great circle same as the sphere itself.

We can also calculate Azimuth

- Use cotangent formulas and the results from the spherical triangle computation.
  - tanA = sinC/((sin(b)/tan(a))-(cos(b)cos(C))
- Note that we must correctly account for the quadrant.
- Note as well that forward and reverse azimuths are not exactly 180 d different.

Other spherical Earth characteristics

- All meridians converge at poles.
- Azimuths of lines measured from one end to not equal values measured from the other end.
  - Effect is especially pronounced on long E-W lines.
- Can be approximated as a function of the E-W distance, mean latitude and spherical radius.

$$\theta'' = \frac{\rho \, \bar{d} \tan \bar{\psi}}{R}$$

Spherical excess

- The summation of all spherical angles exceed 180 degrees.
- It is proportional to the area of the spherical triangle.

$$\varepsilon = \frac{bc \sin A}{2R^2 \sin 1''}$$

```
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+           19 20 21 22 23 24 25 26 27 28 29 30  0  0  0  0  0
+            0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+            0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
+            0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
++           3  3  4  3  3  3  3  3  3  3  3  3  3  3  4  3  3
++           4  3  3  3  3  3  4  3  3  4  3  3  0  0  0  0  0
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++           0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
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%i    0    0    0    0      0      0      0      0         0
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/* cod emr esa gfz jpl mit ngs sio
/* REFERENCED TO IGS TIME (IGST) AND TO WEIGHTED MEAN POLE:
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P  2  20133.793284 -12517.384408  11531.263733    -23.264373
P  3 -11366.447142  22473.599105   7788.691364     63.023929
P  4  26109.950844  -5714.160584   1216.379808    103.694692
P  5 -11013.232261 -21731.530250 -10967.998519    447.812380
P  6 -10108.117501 -15606.106217  19125.583219    170.870195
P  7  16494.168125  -8125.986115 -18697.734751    459.605468
P  8  26002.033551     50.071986   6220.016726    -51.940408
P  9   -872.962985 -15641.294915 -21956.384902      1.771629
P 10   5783.888427 -15080.250565  20946.780855     76.347737
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```

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BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
-Unknown-                                                   MARKER NAME
-Unknown-           -Unknown-                               OBSERVER / AGENCY
-Unknown-           ASHTECH UZ-12       ZE21                REC # / TYPE / VERS
-Unknown-           -Unknown-                               ANT # / TYPE
  -726296.8700 -5598342.1900  2958518.6100                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     7    L1    L2    C1    P2    P1    D1    D2            # / TYPES OF OBSERV
 SNR is mapped to RINEX snr flag value [1-9]                COMMENT
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  L2: 1 -> 1; 150 -> 5; 250 -> 9                            COMMENT
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                                                            END OF HEADER
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    138416.07353     79208.19652  21658215.946    21658212.382    21658216.095
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     -2125.583       -1656.298
    179425.29253     29628.04052  19575418.616    19575412.115    19575414.890
     -2613.179       -2036.243
   -154209.07053   -111215.70852  20094561.440    20094559.983    20094564.353
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   -137048.39353    -93602.12652  17903365.528    17903361.718    17903367.264
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