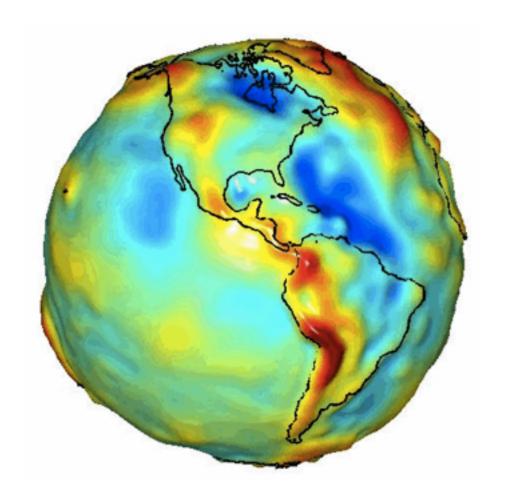
Classes 6 and 7 - Gravity and its importance to Geodesy



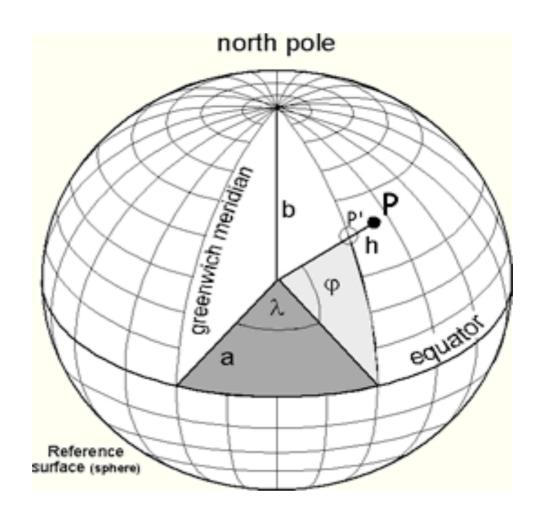
2 and 4 February 2010



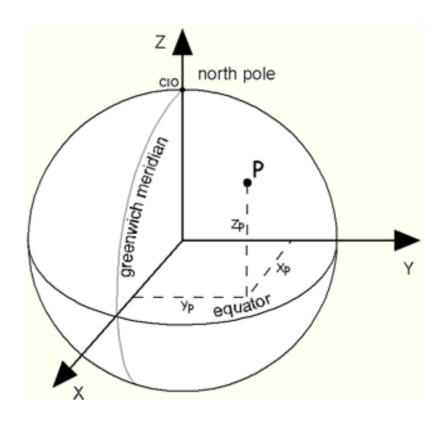
Schedule for this week

- Exam 1 this Thursday during Lab period
- You are responsible for material in Chapters 1-4 in text, homework as well as all lectures and labs to date.
- It is open book and access to the Internet will be needed.

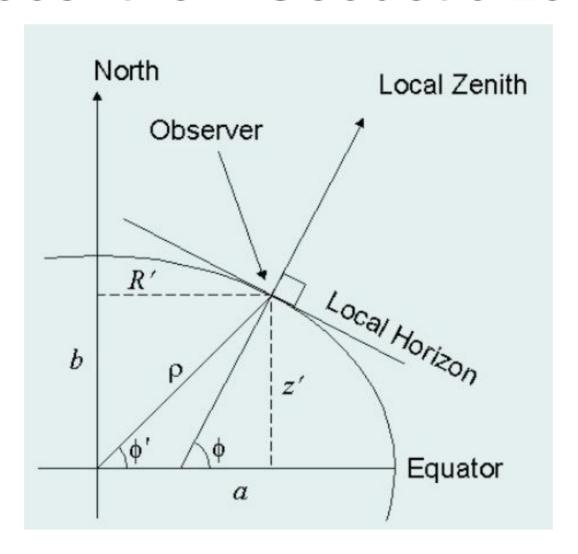
Natural Coordinates



Same Point in XYZ system



Geocentric v Geodetic Latitude



LLHs <-> XYZ

Geocentric relationship to XYZ

 One of the advantages of geocentric angles is that the relationship to XYZ is easy. R is taken to be radius of the sphere and H the height above this radius

$$\phi_c = \tan^{-1}(Z/\sqrt{X^2 + Y^2})$$

$$\lambda_c = \tan^{-1}(Y/X)$$

$$R + H_c = \sqrt{X^2 + Y^2 + Z^2}$$

$$X = (R + H_c)\cos\phi_c\cos\lambda_c$$
$$Y = (R + H_c)\cos\phi_c\sin\lambda_c$$
$$Z = (R + H_c)\sin\phi_c$$

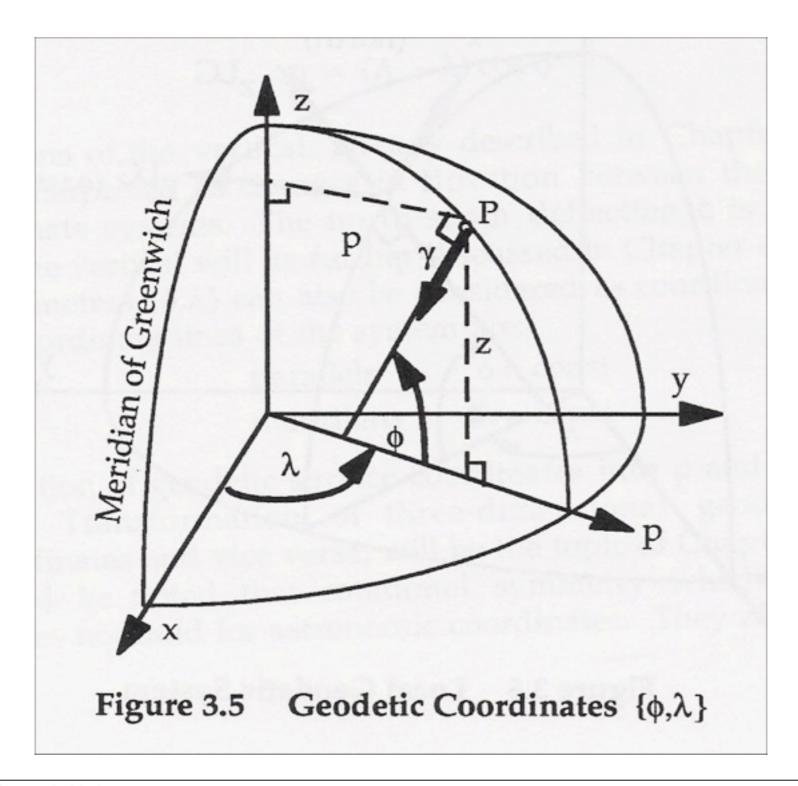
09/18/2006

12.215 Modern Naviation L02

5

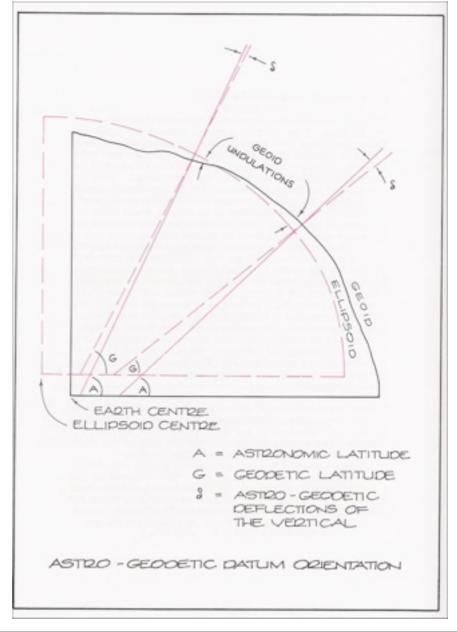
Problem with Geocentric

- Geocentric measures are easy to work with but they have several serious problems
- The shape of the Earth is close to an bi-axial ellipsoid (i.e., an ellipse rotated around the Z-axis)
- The flattening of the ellipsoid is ~1/300
 (1/298.257222101 is the defined value for the GPS ellipsoid WGS-84).
- Flattening is (a-b)/a where a is the semi-major axis and b is the semi-minor axis.
- Since a=6378.137 km (WGS-84), a-b=21.384 km



The System of Natural Coordinates

- Axes are defined by meaningful directions: the gravity vector and of the spin axis of the Earth.
- Gravity vector defines the up-down direction
 - Orthogonal to a level surface.
- There is a difference between the gravity vector and normal to ellipsoid.



Geodetic Reference System 1980 (GRS80)

adopted by the International Association of Geodesy (IAG) during the General Assembly 1979. Principal parameters are:

parameter	symbol	value
defining of	constants	
equatorial radius of the Earth	a	6378137 m
geocentric gravitational constant (including the atmosphere)	GM	$3986005 \cdot 10^8 m^3 s^{\cdot 2}$
dynamical form factor (excluding permanent tides)	\mathbf{J}_2	108263 · 10 ⁻⁸
angular velocity of the Earth	w	$7292115 \cdot 10^{-11} \ rad \ s^{-1}$
derived geometr	rical parame	eters
semiminor axis (polar radius)	b	6356752.3141 m
first excentricity	e^2	0.00669438002290
flattening	f	1:298.257222101
mean radius	\mathbf{R}_1	6371008.7714 m
radius of sphere with same surface	\mathbb{R}_2	6371007.1810 m
radius of sphere with same volume	\mathbb{R}_3	6371000.7900 m
derived physic	al paramet	ers
normal potential at ellipsoid	\mathbf{U}_0	62636860.850 m ² s ⁻²
Normal gravity at equator	g _e	9.7803267715 m s $^{\text{-}2}$
Normal gravity at pole	\mathbf{g}_{p}	9.8321863685 m $\rm s^{\text{-}2}$
wb 09/1999		Comments & suggestion

Universal Law of Gravitation

- Newton formulated the law (1687) to reflect the attraction of two point masses separated by a distance.
 - $f = G^* [(m^*m')/l^2]$
 - (f is force, m and m' are point masses, l is distance and G is Newton's gravitational constant)
- Currently accepted value for G
 - $-6.67259 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$

More mind-numbing detail...

- At latitude p, a conventional value of the acceleration of gravity at sea level is given by the International Gravity Formula,
 - $-g = 978.0495 [1 + 0.0052892 sin^2(p) 0.0000073 sin^2 (2p)] cm per second per second (cm s⁻²).$
- The mean Earth gravity is about 981 000 mGal (the well-known 9.81 *m*/s²), varies from 978,100 mGal to 983,200 mGal from Equator to pole due to the Earth's flattening and rotation.

Gravitational Acceleration

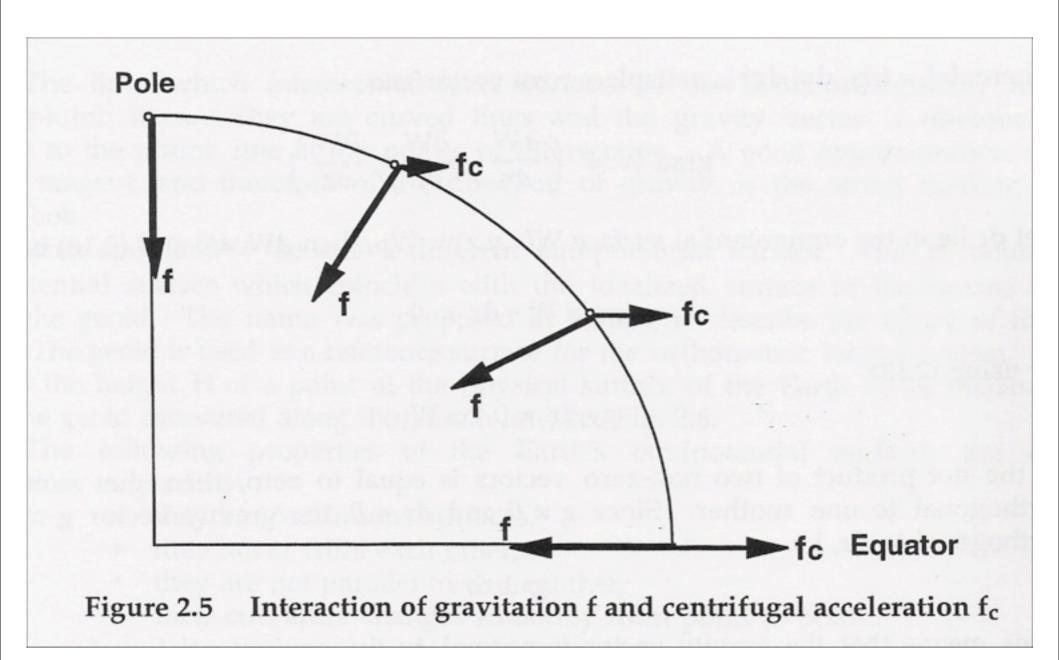
 The magnitude of acceleration (b) due to the Earth's mass on the surface using a spherical geometric reference surface (R) is:

$$-b = GM/R^2$$

```
>> b = 398600.5e9/6371000^2
b =
9.8203
```

Centrifugal Acceleration

- Direction is always perpendicular outward from the spin axis.
- It is a function of angular velocity of the Earth squared and the distance from the point of interest on the surface of the sphere to the axis of rotation.
- $\omega = 7292115e-11rad sec-1$
- We can compute this value as ratio of degrees over time.



Gravitational Attraction

- Is the vector sum of gravitational and centrifugal acceleration.
- The actual acceleration of gravity varies from place to place, depending on latitude, altitude, and local geology.
- By agreement among physicists, the standard acceleration of gravity (gn) is defined to be exactly 9.80665 meters per second per second (m s-2), or about 32.174 05 feet per second per second.

Magnitude of centrifugal acceleration

- Varies from equator to poles.
- Compute magnitude by velocity squared times the distance from the point of interest to the spin axis.

```
>> omega = 7292115e-11;
>> R = 6371000;
>> CentrifAccelEquator = omega^2*R

CentrifAccelEquator =

0.0339
```

Gravity and Geodesy

- Defines a plumb line (local vertical) defined by gravity.
- Gravity also serves as an important reference surface. It is the level surface that is perpendicular to the plumb line at all points.

Gravitational Potential

- Magnitude of the potential is the work that must be done by gravity to move a unit mass from infinity to the point of interest.
- Is dependent on position within the gravitation field.

Equipotential Surfaces

- Surface having constant gravity potential.
 - Also known as level surfaces or geopotential surfaces.
- Surfaces are perpendicular at all points of the plumb line (gravity vector).
- Potential field is a scalar field from which the vector gravity field can be found.
- A still lake surface is an equipotential surface.
 - It is not horizontal but curved.

Properties of equipotential surfaces

- They are closed continuous surfaces that never cross one another.
- They are formed by long radius arcs.
 Generally without abrupt steps.
- They are convex everywhere.

$$\int_{a}^{p} \vec{f} d\vec{l}$$

 \vec{f} is the force dependent upon position within the force field. $d\vec{l}$ is an element of distance. Note that the potential is expressed in units of work and that it is path independent.

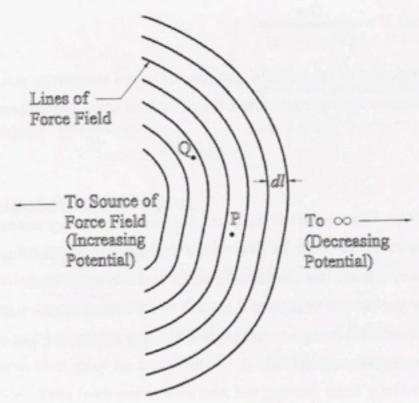
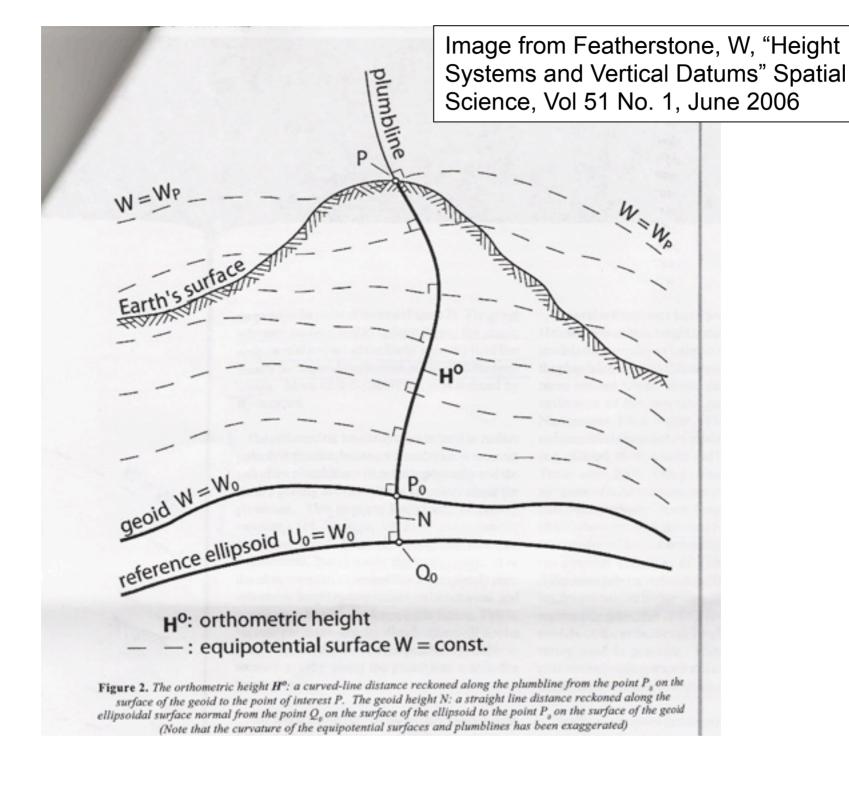
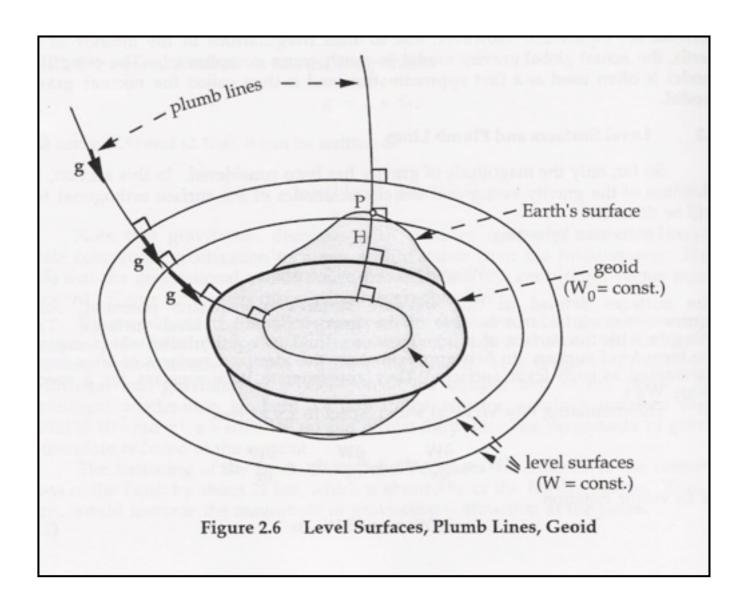


Figure 4.2 Positions of Points P and Q in a Force Field.

The change in potential (work required to move) from point P to point Q within a force field is:





Gravity defines the plumb line – the local vertical

What is Geodesy?

- "Geodesy is the discipline that deals with the measurement and representation of the earth, including its gravity field, in a threedimensional time varying space."
 - definition adopted by the National Research Council of Canada in 1973. (Vanicek, P.K. and Edward Krakiwsky, E.(1986) Geodesy: The Concepts. Elsevier).

Another definition

- "The task of geodesy is the determination of the potential function W(x,y,z)" i.e. of the gravity potential of the Earth.
 - By Heinrich Bruns (1878)
- Both definitions indicate the linkages between positioning and gravity field determination.

Integrated Geodesy

- Also called "Operational Geodesy"
- Integrated geodesy is a method in which a wide variety of surveying measurements are modeled in terms of geometric positions and the earth's geopotential.
 - Both geometric and gravimetric data are simultaneously estimated using Least Squares.

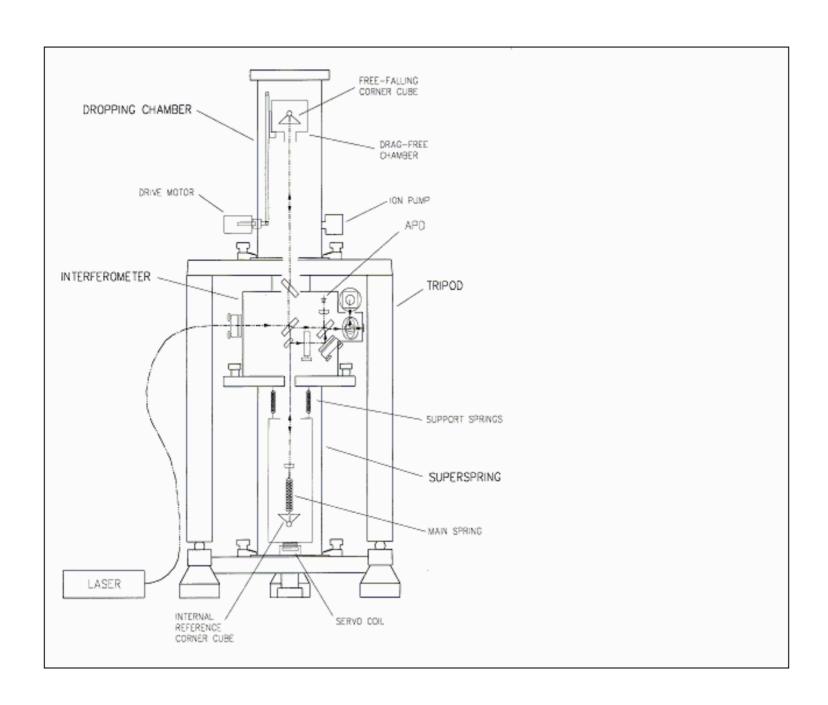
Absolute Gravity Meters





How does it work?

- Motion of a test mass free falling in a vacuum is interferometrically measured with respect an inertial reference.
- Controlled carriage assembly releases the test mass (a corner cube retroreflector mounted in an aluminum housing).
- The inertial reference is another corner cube retroreflector mounted on a force feedback long period (60sec) seismometer.
- Non-gravitational forces are minimized (air drag, electrostatics, and eddy current damping).



Gravity change

- FG5 accuracy:
 - Instrument 1.1 μGals
 - Environmental: 1.5 μGals
 - •Observational error: ~0.4 μGals
 - •RMS of above at instrument height (131 cm): 1.9 µGals
 - •RMS with relative transfer to mark or excenter: 3 to 8 μGals

- +3 μGals corresponds to
 - -1 cm elevation change
 - $+7\frac{1}{2}$ foot rise in water-table
 - GPS to resolve ambiguity
- Can also measure
 - magma insertion
 - sea level change (with tide record comparisons)
 - glacial ice mass change

ERIE CU (11-12 Aug 98) MAG-1, Reference Ht. = 100 cm

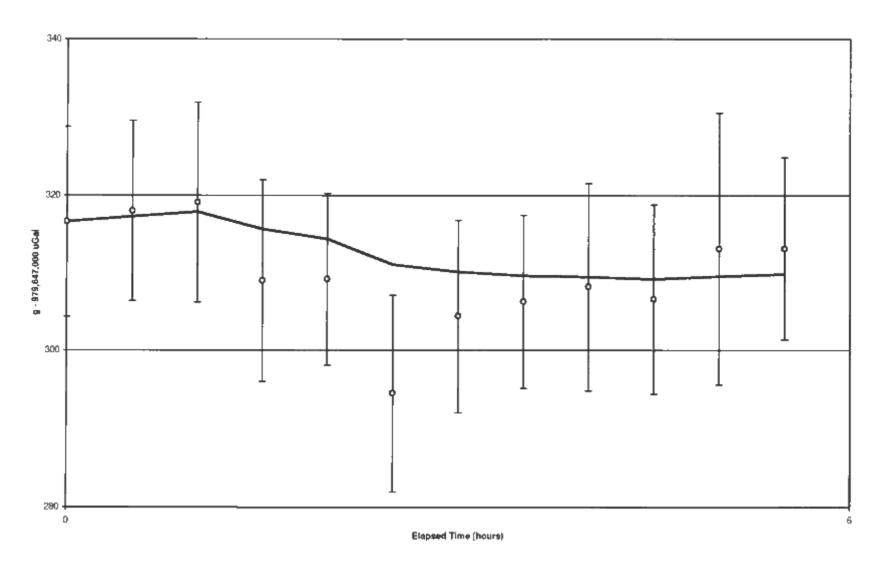
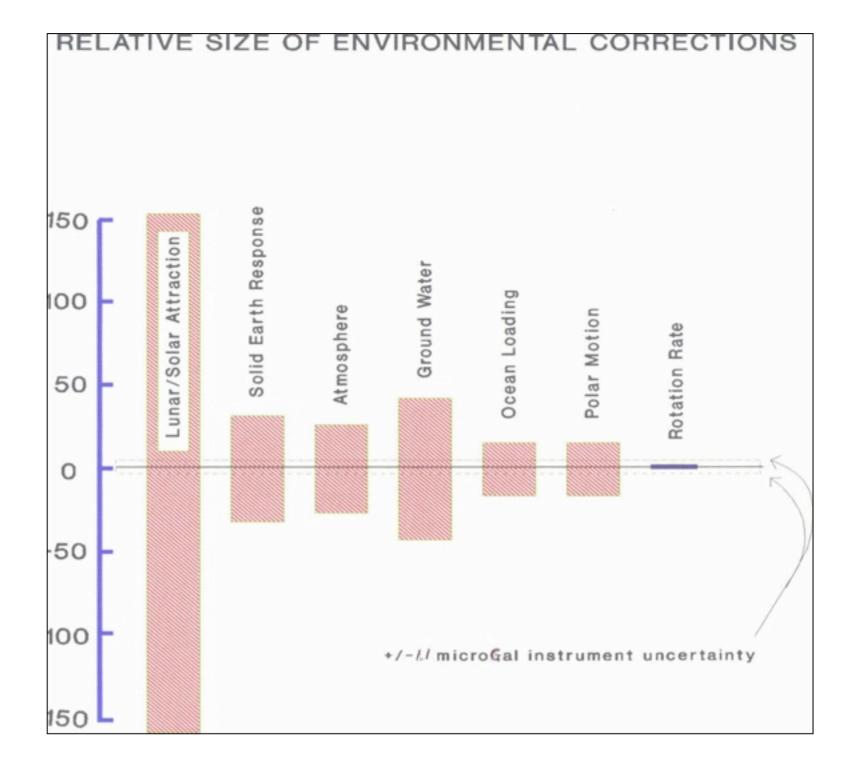
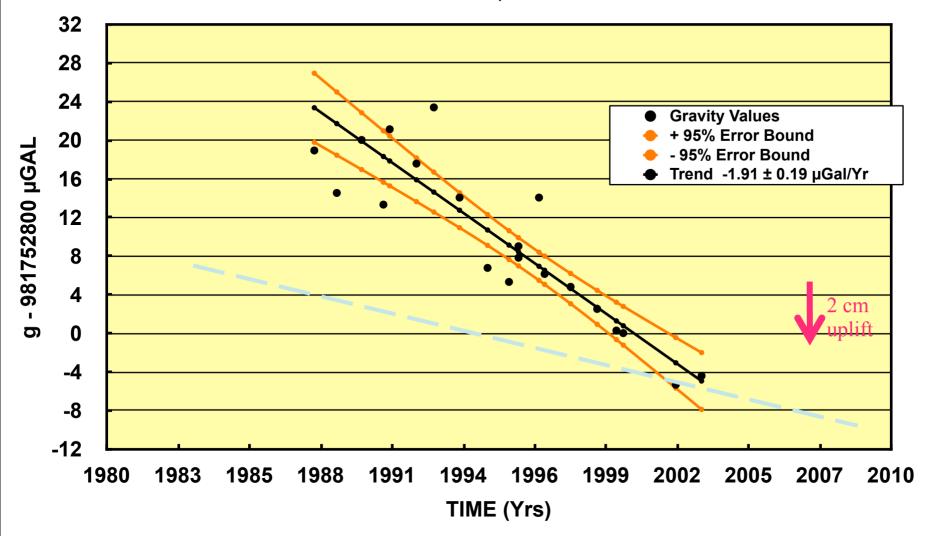
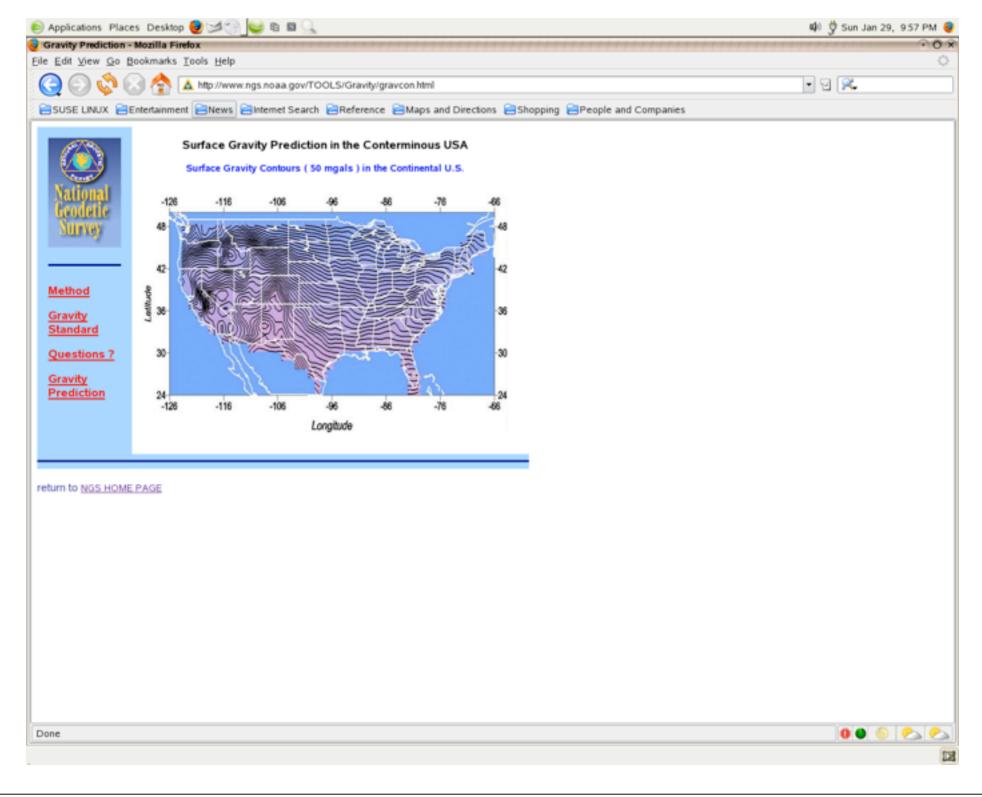


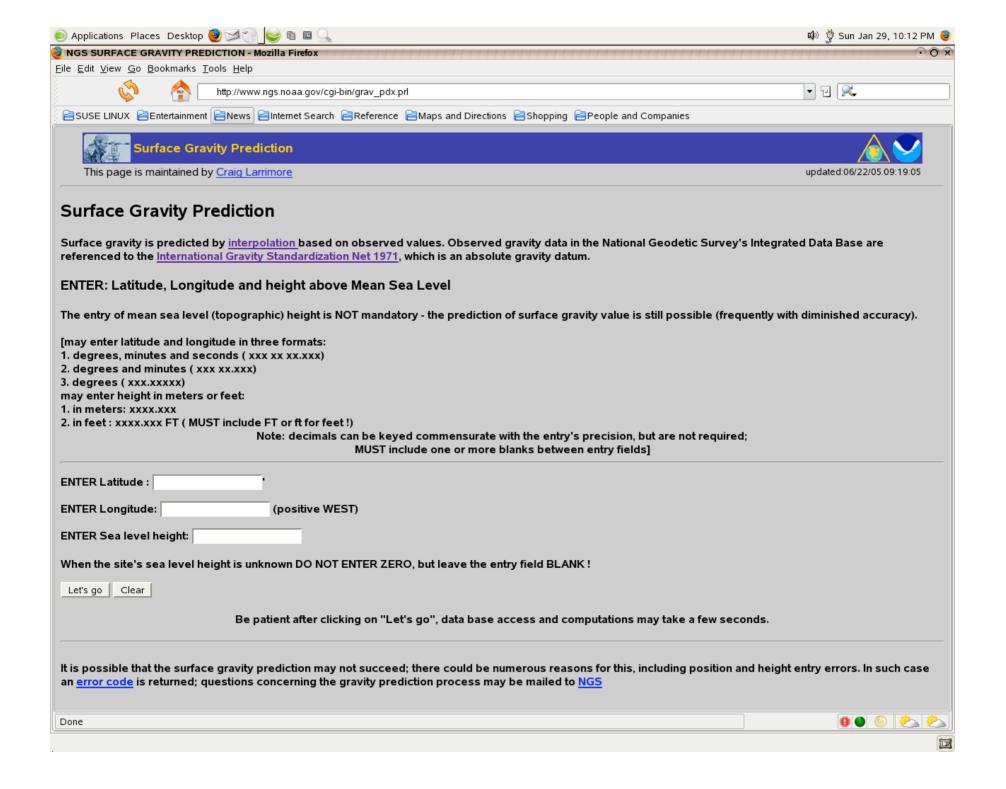
Figure 3



CHURCHILL, MANITOBA







```
The predicted gravity is given in units of 'milligals':
  1 gal
             = 1 cm/sec2 ('gal' in honor of Galileo)
  1 kilogal = 1000 gal
  1 milligal = 1/1000 gal
Questions concerning the gravity prediction process may be mailed to NGS
Latitude: 27 42 52.08857
Longitude: 097 19 44.31265
MSL Height: 5
Predicted gravity: 979129 +/- 2 milligals
```

NAVD 88 Gravity Computations

Use this program to compute NAVD 88 surface gravity as a function of latitude and longitude.

Gravity is computed by the method used to compute gravity for the NAVD 88 adjustment.

This is also the method used to compute the Modelled Gravity which shows on NGS data sheets.

Computations may be performed for a specific geographical location or for a file of input points.

The NAVD 88 gravity may differ from the measured gravity at a point, or from the gravity value produced by the <u>Surface Gravity tool</u> in this toolkit by a few milligals. This is because:

- The interpolation method used in computing NAVD 88 gravity may introduce some smoothing of measured values.
- NAVD 88 gravity is computed using a gravity model determined at the time of the NAVD 88 adjustment.
 New or revised values added to the NGS gravity data base since that time may affect the Surface Gravity computations but will have no effect on the NAVD 88 gravity.

Compute NAVD 88 gravity for a specific location:

Enter North Latitude and West Longitude below.

All values must be entered as integers (no decimals).

The NAVD 88 gravity model is valid only within the coterminous United States.

Degrees Minutes Seconds(integer only)

Latitude 27 42 52

Longitude 097 19 44

Compute NAVD 88 Gravity for a Single Location

Clear Form

Output from NAVD Gravity Computations

NGS program grav info (formerly GETGRAV) version: 2.3 date: 01/04/25

latitude longitude gravity(gals) std. dev.(mgal) 274252 0971944 979.1280 2.7



The predicted gravity is given in units of 'milligals':

1 gal = 1 cm/sec² ('gal' in honor of Galileo)

1 kilogal = 1000 gal

1 milligal = 1/1000 gal

Questions concerning the gravity prediction process may be mailed to NGS

Latitude: 27 42 52.08857

Longitude: 097 19 44.31265

MSL Height: 5

Predicted gravity: 979129 +/- 2 milligals

Output from NAVD Gravity Computations

NGS program grav_info (formerly GETGRAV) version: 2.3 date: 01/04/25

latitude longitude gravity(gals) std. dev.(mgal) 274252 0971944 979.1280 2.7



Which model to use?

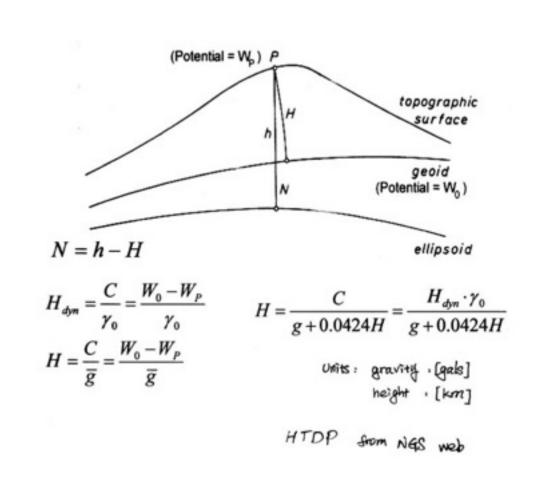
 NAVD88 - Modeled Gravity uses a model developed for the NAVD88 adjustment rather than current gravity values.

Review of Height Systems

- Helmert Orthometric
- NAVD 88

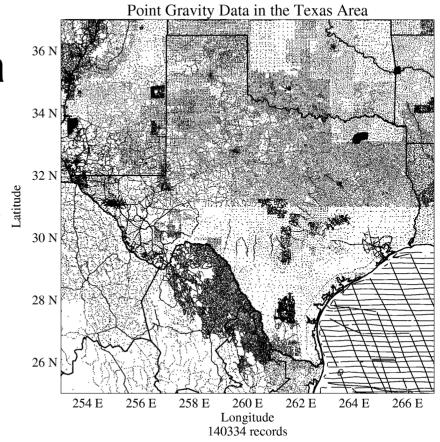
$$H = \frac{C}{\overline{g}} = \frac{C}{g + 0.0424H}$$
$$\overline{g} = g - \left(\frac{1}{2}\frac{\partial \gamma}{\partial h} + 3\pi G\rho\right)H$$

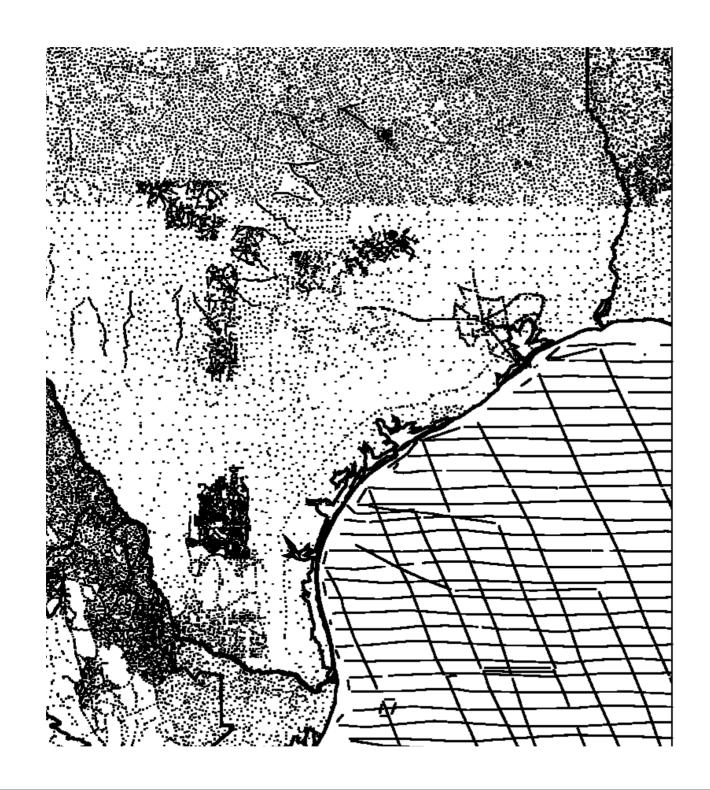
- local gravity field (g)
- single datum point
- follows MSL



Earth's Gravity Field from Space

- Satellite data was used for global models
 - Only useful at wavelengths of 700 km or longer
- Lower wavelength data from terrestrial or marine gravity of varying vintage, quality and geographic coverage





Gravity

- Static gravity field
 - Based on long-term average within Earth system
- Temporally changing component
 - Motion of water and air
 - Time scale ranges from hours to decades.
- Mean and time variable gravity field affect the motion of all Earth space vehicles.

Geoid

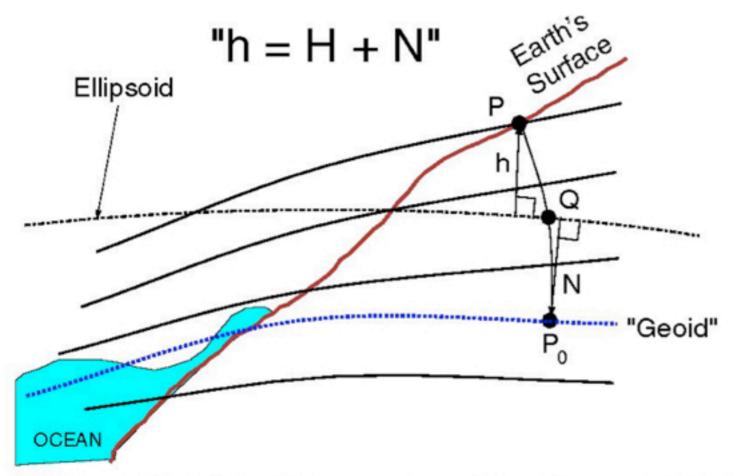


 ζ = Mean Ocean Dynamic Topography

Even after time-averaging, Local Mean Sea Level does not correspond to a level surface.

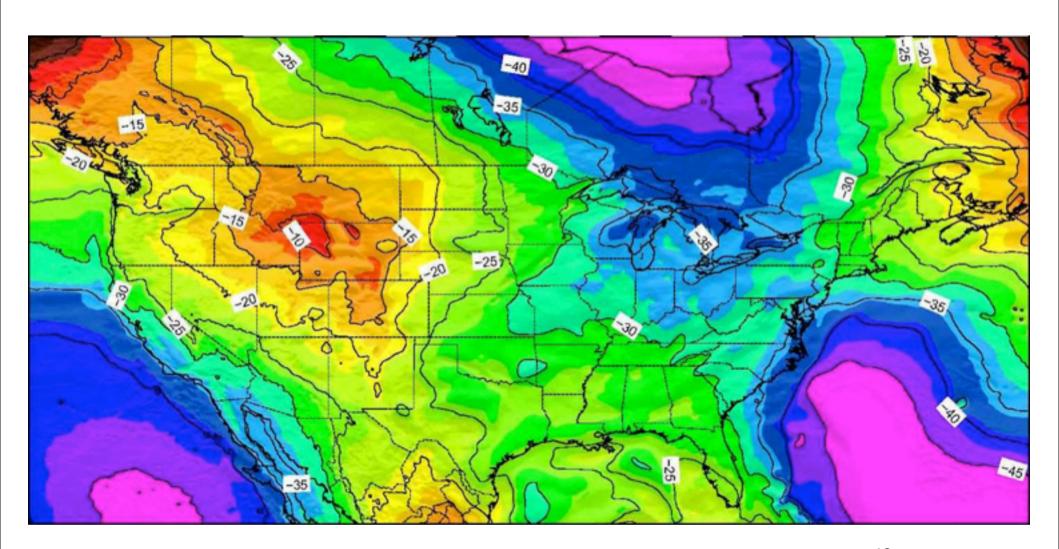
 ζ is induced by permanent ocean circulation patterns, salinity, temperature and wind set-up, which vary geographically.

 The equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level.



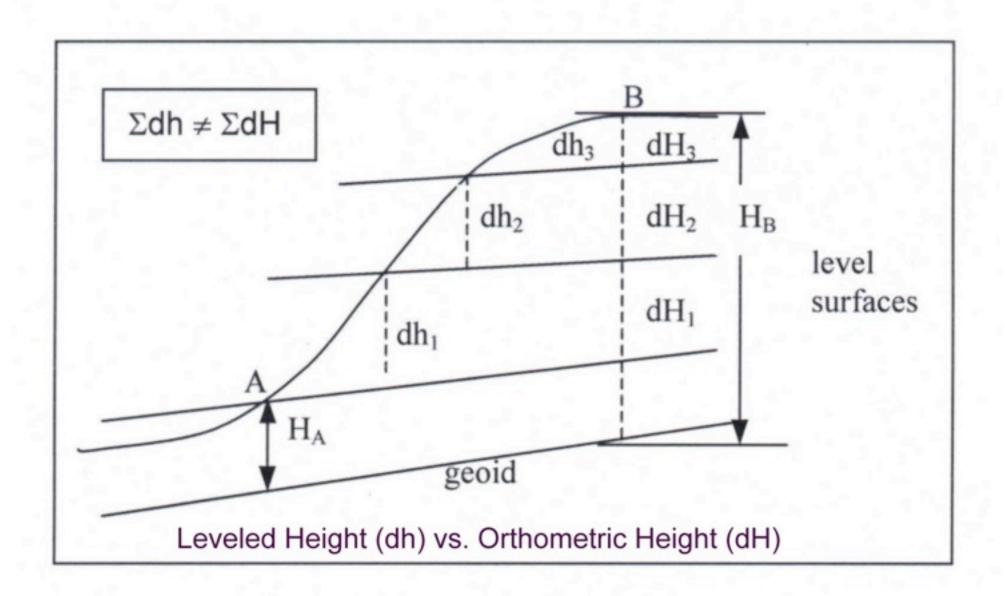
h (Ellipsoid Height) = Distance along ellipsoid normal (Q to P) N (Geoid Height) = Distance along ellipsoid normal (Q to P_0) H (Orthometric Height) = Distance along Plumb line (P_0 to P)

GEOID 2009

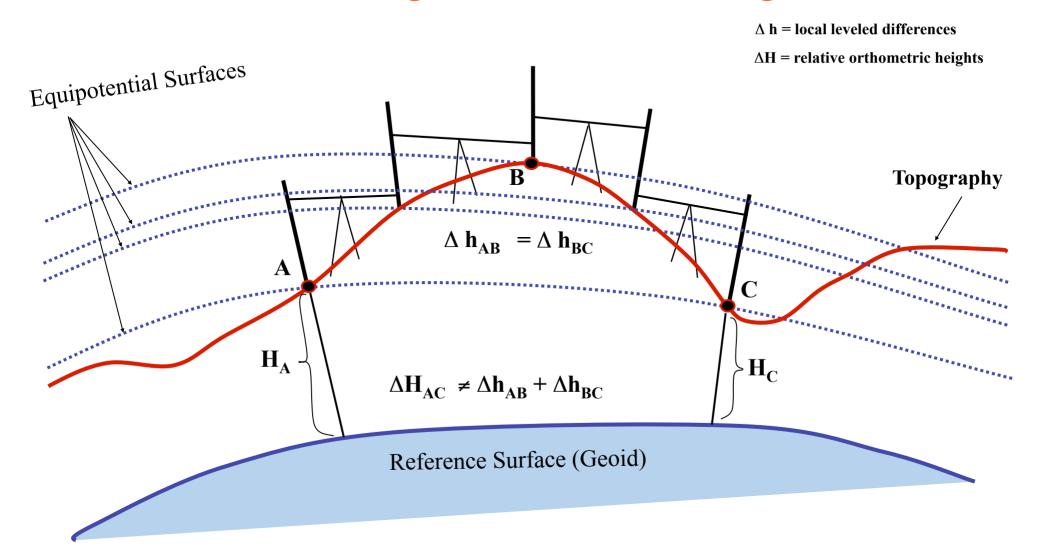


Leveling Issues

- Height
 - The distance measured along a perpendicular between a point and a reference surface.
- Raw leveled heights
 - Non-unique. Depending on the path taken a different height will be determined for the same point. They have NO physical relevance.



Leveled Height vs. Orthometric Height



Observed difference in orthometric height, ΔH , depends on the leveling route.

Adjusted Geopotential Numbers/Elevations by Line

Line/Par Mark ID	t: L263 SSN	56/5 PID	SSN+: Designation	mark float	ed, SSN*:	mark	constrained,	SSN#: mark flo Geopotential	ated & const Elevation	rained Codes
0 1 2	1377* 1376* 1378*	HU0596 HU0597 AI7161	W 105 MOORES AZ MK Y 107 RESET					2.5267 2.3772 2.2908	2.5783 2.4257 2.3376	
3 4	1379 1380*	HU0600 AJ8000	X 105 TULLS					2.6299 2.1135	2.6836 2.1567	
5	1381*	HU0602	z 105					2.7154	2.7709	

N = h - H(Potential = W_p) P topographic surface $geoid (Potential = <math>W_0$) ellipsoid

$$\begin{split} H_{dyn} &= \frac{C}{\gamma_0} = \frac{W_0 - W_P}{\gamma_0} \\ H &= \frac{C}{g} = \frac{W_0 - W_P}{\overline{g}} \end{split} \qquad H = \frac{C}{g + 0.0424H} = \frac{H_{dyn} \cdot \gamma_0}{g + 0.0424H} \\ H &= \frac{C}{\overline{g}} = \frac{W_0 - W_P}{\overline{g}} \end{split} \qquad \text{Units: gravity (gals)}$$

HTDP from NGS web

Dynamic Heights

- Takes the geopotential number at a point and divides it by a constant gravity value.
- The constant value used in the US is the normal gravity value at 45 degrees N latitude
 - -980.6199 gals

CC area Dynamic Heights

```
National Geodetic Survey, Retrieval Date = JANUARY 31, 2007
AC8464 TIDAL BM
                   - This is a Tidal Bench Mark.
AC8464 DESIGNATION - 877 5792 A TIDAL
AC8464 PID

    AC8464

AC8464 STATE/COUNTY- TX/NUECES
AC8464 USGS QUAD - CRANE ISLANDS NW (1975)
AC8464
AC8464
                             *CURRENT SURVEY CONTROL
AC8464
AC8464* NAD 83(1986) - 27 38 05. (N)
                                        097 14 15.
                                                        (W)
                                                                SCALED
                                                 2.76 (feet) ADJUSTED
AC8464* NAVD 88
                            0.840 (meters)
AC8464
AC8464 GEOID HEIGHT-
                            -25.91
                                                                GEOIDO3
                                    (meters)
                              0.839 (meters)
                                                  2.75
AC8464 DYNAMIC HT -
                                                                COMP
                                                        (feet)
AC8464 MODELED GRAV- 979,127.0
                                    (mgal)
                                                                NAVD 88
AC8464
                               CLASS II
AC8464 VERT ORDER - FIRST
```

NAVD 88 and dynamic heights differ by only 1 mm at this station.

Dynamic Heights

```
DATABASE = Sybase , PROGRAM = datasheet, VERSION = 7.42
        National Geodetic Survey, Retrieval Date = JANUARY 31. 2007
AC5717 DESIGNATION - 7850
ACS717 PID
                  - AC5717
AC5717 STATE/COUNTY- TX/JEFF DAVIS
AC5717 USGS QUAD - MOUNT LOCKE (1978)
AC5717
 AC5717
                             *CURRENT SURVEY CONTROL
AC5717
AC5717* NAD 83(1993) - 30 40 49.77247(N) 104 00 54.30121(W)
                                                              ADJUSTED
AC5717* NAVD 88 - 2026.545 (meters) 6648.76 (feet)
                                                              ADJUSTED
AC5717
AC5717 X
                - -1,330,007.642 (meters)
                                                              COMP
                - -5,328,393.026 (meters)
AC5717 Y
                                                              COMP
            - 3,236,502.804 (meters)
AC5717 Z
                                                              COMP
AC5717 LAPLACE CORR-
AC5717 ELLIP HEIGHT-
                            -0.25 (seconds)
                                                              DEFLEC99
                         2005.36 (meters)
                                                   (10/23/00) GPS OBS
AC5717 GEOID HEIGHT-
                          -21.19 (meters)
                                                              GEOIDO3
AC5717 DYNAMIC HT - 2023.109 (meters)
                                              6637.48 (feet) COMP
AC5717 MODELED GRAV- 978,871.0
                                                              NAVD 88
                                   (mgal)
 AC5717
AC5717 HORZ ORDER - B
                                               NAVD 88 = 2026545 \text{ m}
AC5717 VERT ORDER - FIRST CLASS II
 AC5717 ELLP ORDER - FOURTH CLASS II
                                               DYN Hght= 2023.109
                                               Difference = 3.436 \text{ m}
```



This page is maintained by NGS Software Requests

This utility computes a dynamic height based on the input NAVD 88 orthometric height and gravity value For more information about dynamic heights see file dynamic ht.doc

Enter a surface gravity value in units of mgal or a latitude and longitude to have the program automatically interpolate a gravity value.

The latitude and longitude on the NAD 83 datum has the following permissable formats:

- degrees and decimals (xx.xxxx)
- · degrees, minutes and decimals (xx xx xxxxx)
- · degrees, minutes, seconds and decimals (xx xx xx xxxx)

The input elevation should be in meters and decimals.

Surface Gravity:	-OR- Latitude: 30 40 49.8 Longitude: 104 00 53.9	
NADV88 Helm	ert Orthometric Height: 2026.545	

Press 'Submit' to compute a dynamic height for the above input.

	Submit	Clear	Sample1	Sample2	Sample3	Sample4
--	--------	-------	---------	---------	---------	---------

Return to GEODETIC TOOLKIT PAGE Return to NGS HOME PAGE

Dynamic Height Problems

 Dynamic height corrections applied to spiritleveled height differences can be very large (several meters) if the chosen gravity value is not representative for the region of operation.

Orthometric correction

- Occurs because equipotential surfaces are not parallel.
- In general, not parallel in a N-S direction but they are parallel in an E-W direction.
- It is computed as a function of mean orthometric height and latitude of section.

Leveled v Ortho. Height Diffs

- Due to non-parallelism of level surfaces the differences between published NAVD 88 points do not correspond to the leveled height difference.
- Therefore the subtraction of the published NAVD 88 heights on the NGS datasheets will NOT agree with the leveled difference.
- This is problem mostly in high elevations.



This page is maintained by NGS Software Requests

Compute the expected leveled height differences between two NAVD 88 bench marks

This utility uses NGS program <u>LVL DH</u> to compute height differences between two NAVD 88 bench marks by one of the two methods below.

For more information about the computation see file lvl dh.doc

RETRIEVE BY VALUES...

Clear or overwrite the Orthometric Height and Gravity Value for the two MARKS below, then click Submit to compute results.

MARK #1 MARK #2

Ortho. Ht. = 500.000 525.333

Gravity = 978222.000 978225.444

Submit Clear Sample

2. RETRIEVE BY PIDs...

An example from Colorado

- The difference between published NAVD 88 heights is 402.156 m.
- The leveled difference is 402.085 m.

```
Level Height Difference Computation
LVL_DH Program
Version 2.1

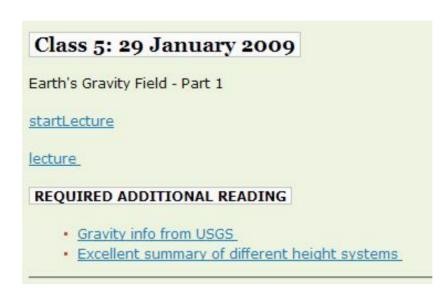
Computation #: 1

HEIGHT 1 GRAV 1 HEIGHT 2 GRAV 2 LEVEL DIFF.
(meters) (mgal) (meters) (mgal) (meters)

2659.349 979133.2 3061.505 979074.9 402.085
```

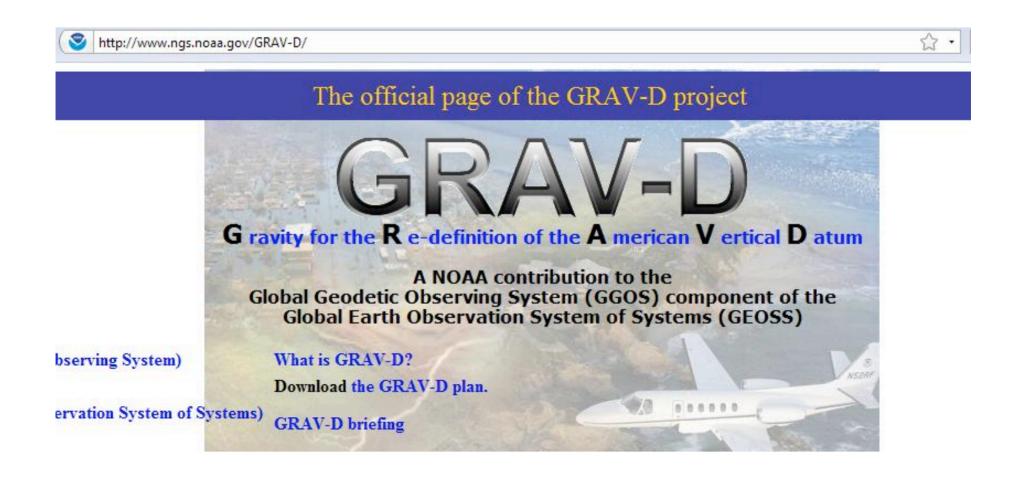
Reminder

 You are responsible for the material listed as REQUIRED ADDITIONAL READING in the Class 5 section of the web page.

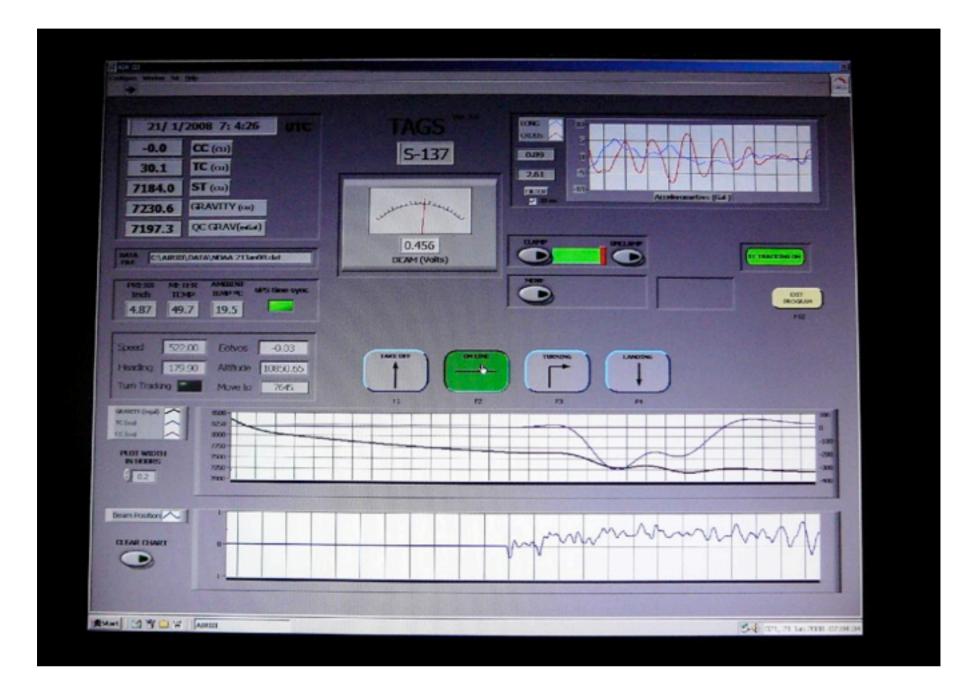


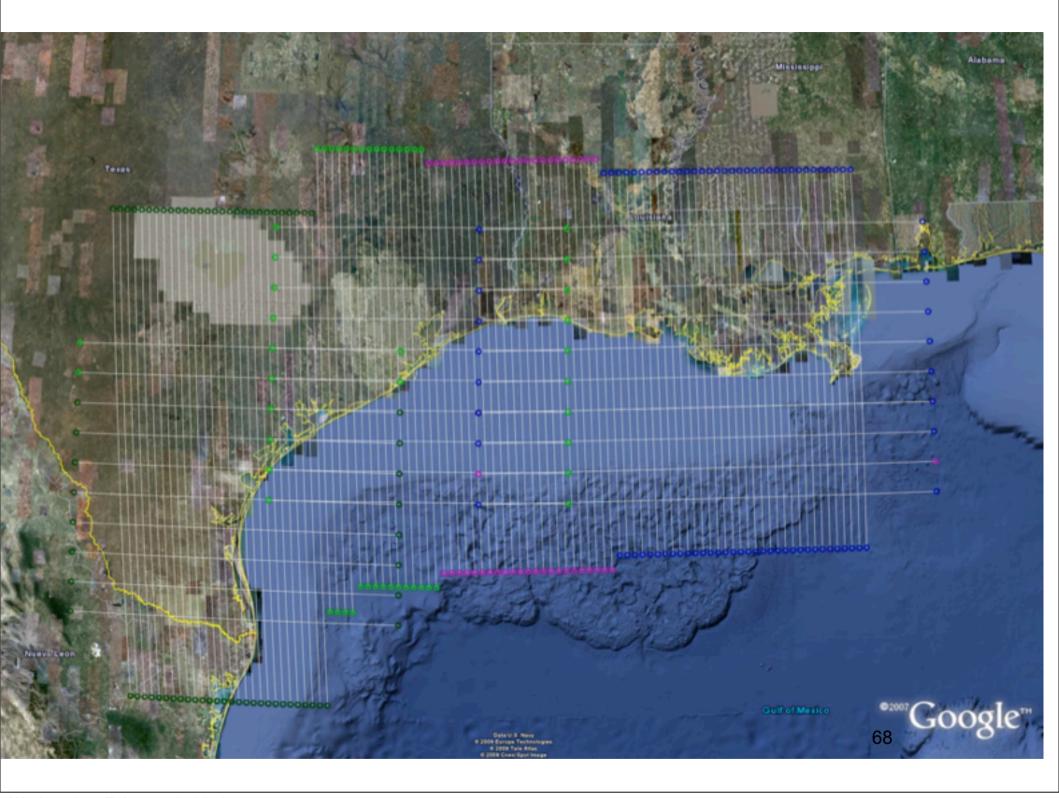


http://www.ngs.noaa.gov/GRAD-D









Gravity field and steady-state Ocean Circulation Explorer

Description

The GOCE gradiometer contains three pairs of proof masses positioned at the outer ends of three 50 cm long orthogonal arms. Because of their different position in the gravitational field they all experience the gravitational acceleration of the Earth slightly differently. The three axes of the gradiometer allow the simultaneous measurement of six independent but complementary components of the gravity field.

Facts and figures



Full name: Gravity field and steady-state Ocean Circulation Explorer

Launch: March 2009

Launcher: Rockot (with Breeze-KM upper stage) by Eurockot Launch

Services GmbH

Launch site: Plesetsk Cosmodrome, Russia

Mission control: European Space Operations Centre (ESOC),

Darmstadt, Germany

Number of instruments: 3

Nominal life: 20 months

Mission cost: €340 million (including launcher and operations)

Orbit: about 260 km altitude, polar, Sun-synchronous

Mass: 1100 kg

Size: 5.3 m long, about 1 m body diameter

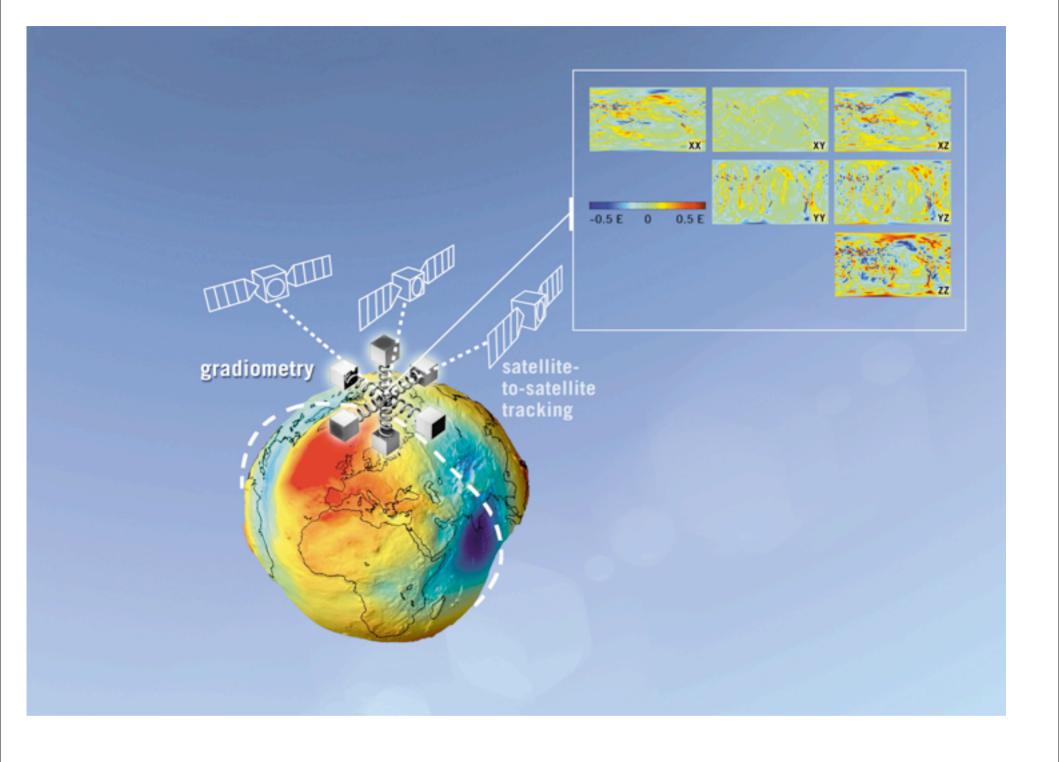
Propulsion tank: 40 kg of xenon

Geoid accuracy: 1 - 2 cm vertically with 100 km spatial resolution

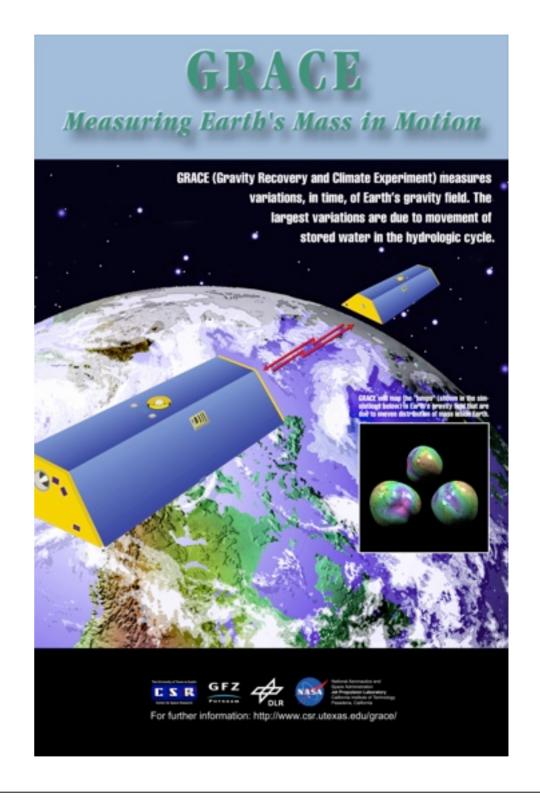
Gravitational acceleration at Earth's surface is about 9.8 m/s², varying from a minimum of 9.788 m/s² at the equator to a maximum of 9.838 m/s² at the poles.

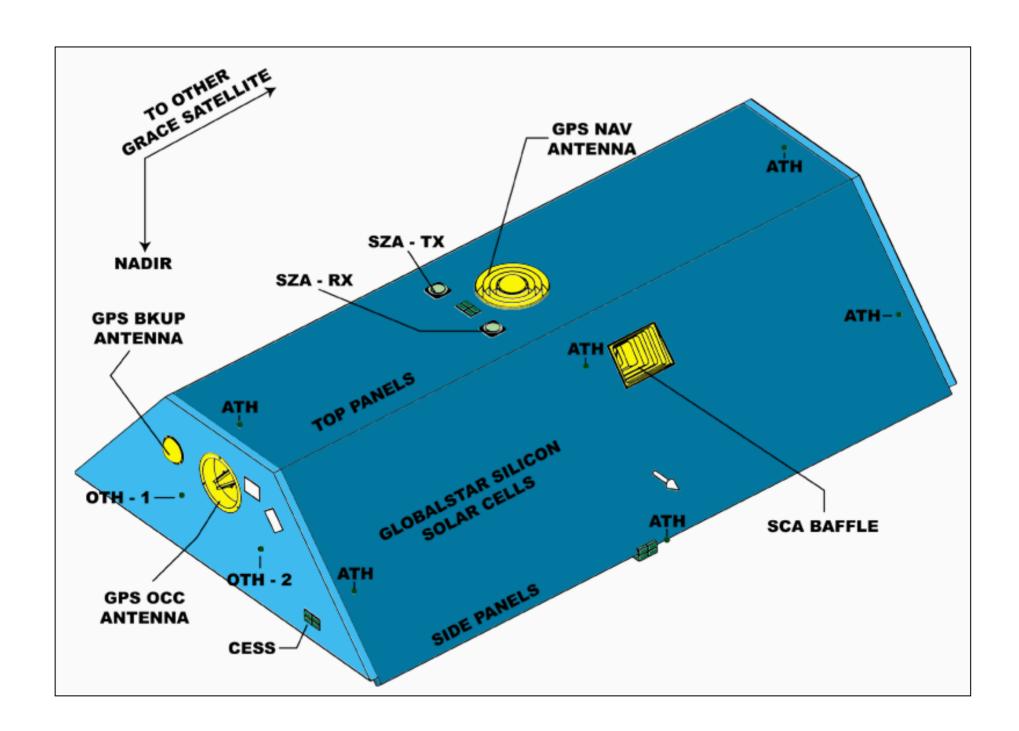
The six accelerometers (three pairs in three orthogonal directions) are some 100 times more sensitive than any previously flown in space.

Planning and construction of the GOCE spacecraft involved 45 European companies led by Thales Alenia Space.

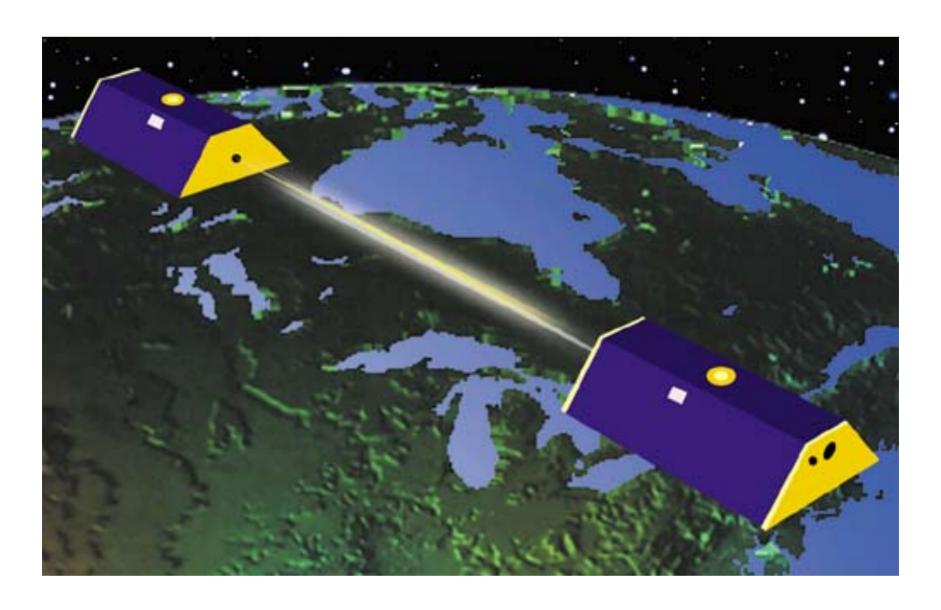


Gravity
Recovery
And
Climate
Experiment





GRACE

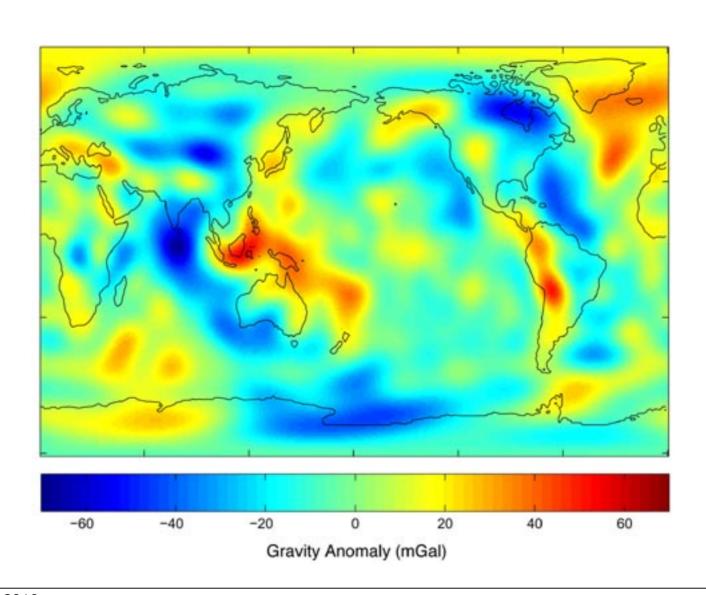


How does GRACE work?

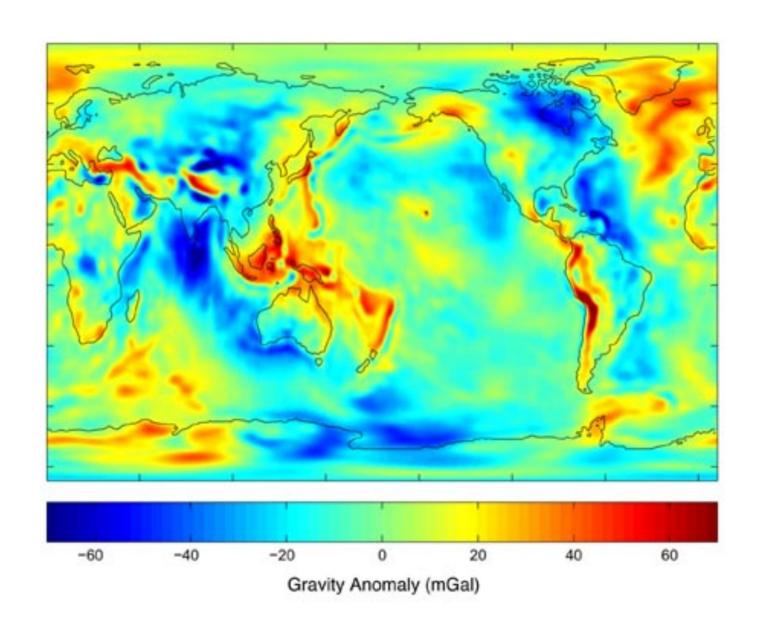
- Motion of two satellites differ because they are at different positions in space.
- When the lead SV approaches a higher gravity mass it accelerates as it moves beyond it decelerates.
- Distance changes between SVs is measured precisely.



Geoid Model from Earth Orbiting Space Vehicles (pre-GRACE)



GRACE 111 days of data



GRACE 363 days of data

