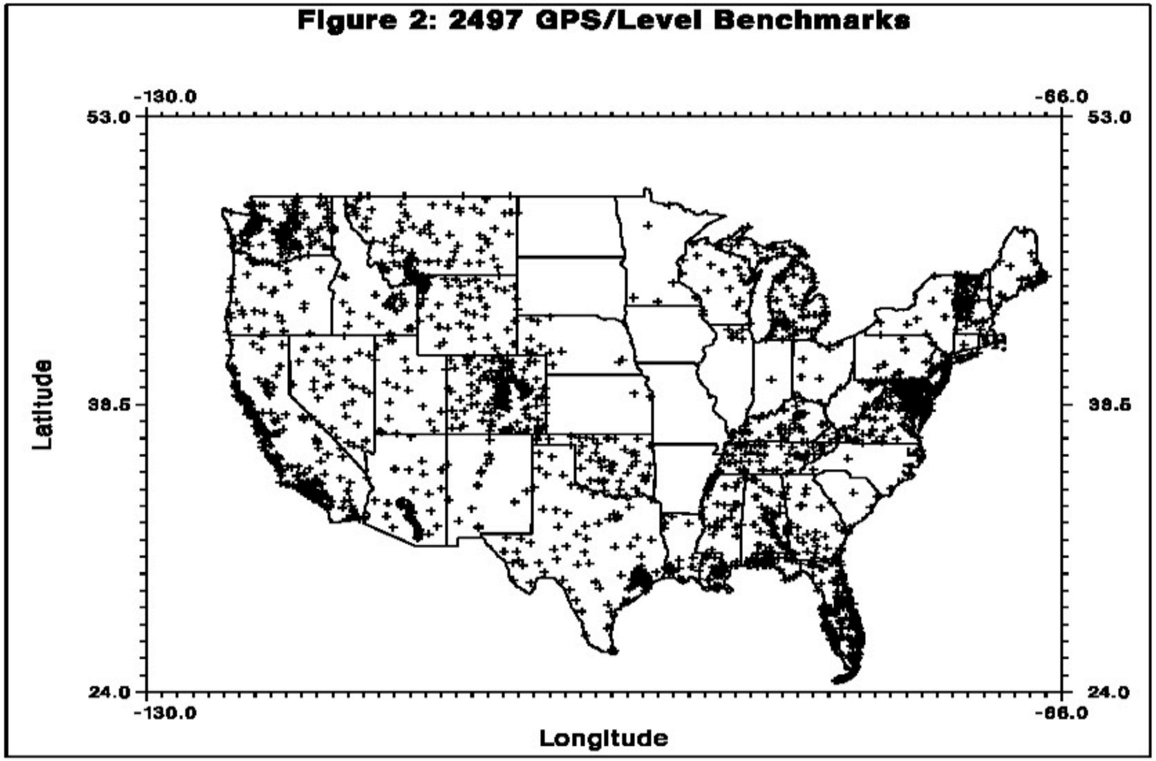
Lecture 15: Vertical Datums and a little Linear Regression



For Geoid96

GISC-3325

4 March 2010

Update

- Reading for next two classes
  – Chapter Seven (lots of lovely computations)
- An extra-credit opportunity is available. Details are posted to class web page.
- Other ideas should be discussed with Instructor.

What is a vertical datum?

- Example: North American Vertical Datum of 1988 (NAVD 88)
- ***Definition:*** The surface of equal gravity potential to which orthometric heights shall refer in North America*, and which is 6.271 meters (along the plumb line) below the geodetic mark at "Father Point/Rimouski" (NGSIDB PID TY5255).
- ***Realization:*** Over 500,000 geodetic marks across North America with published Helmert orthometric heights, most of which were originally computed from a minimally constrained adjustment of leveling and gravity data, holding the geopotential value at "Father Point/Rimouski" fixed.

Vertical datums in the USA

- **NGVD 29**
  - National Geodetic Vertical Datum of 1929
  - Original name: "Sea Level Datum of 1929"
  - "Zero height" held fixed at 26 tide gauges
  - Did not account for Local Mean Sea Level variations from the geoid
    - Thus, not truly a "geoid based" datum

Global Sea Level

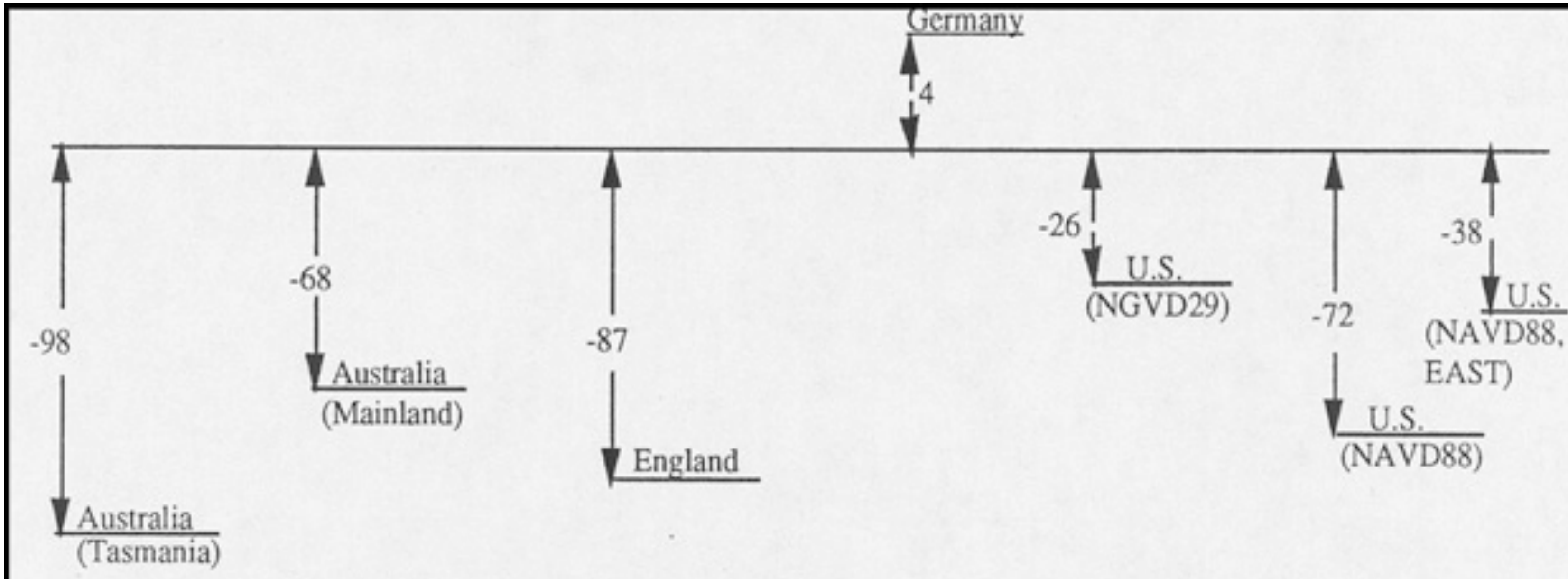
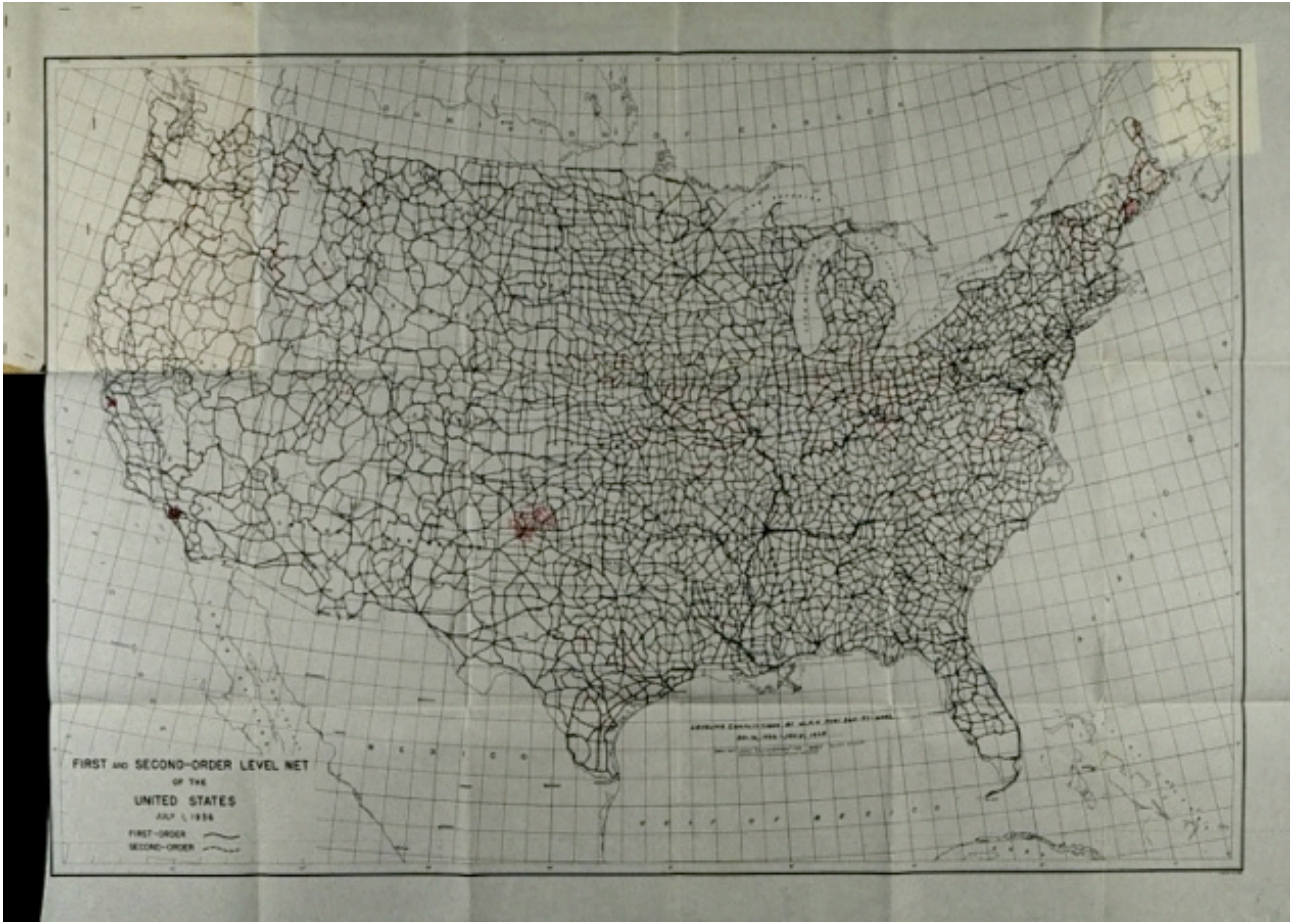


Fig. 1. Summary of Reference Surface Differences with Respect to the Geoid. Units are cm.

NGVD 29

- Defined by heights at 26 tide stations in the US and Canada.
- Tide Gages connected to the vertical network by leveling.
- Water-level transfers were used to connect leveling across the Great Lakes.
- Used normal orthometric heights
  - scaled geopotential numbers using normal gravity

VERTICAL CONTROL USED IN 1929 ADJUSTMENT



First and Second-order Level network as of 1936

Problems with NGVD 29

PROBLEMS WITH THE PRESENT NGVD 29 SYSTEM

Problems	Approx. Amount (Meters)	Corrections New Surveys	Corrections New Adjust.
Adding 1,225K km to Old 75K km Network	0.3		X
Constraining Tide Gage Heights to NGVD 29	0.7		X
Lack of Gravity Data in NGVD 29 Adjustment	1.0	X	X
Refraction Errors	0.5		X
Post-Glacial Uplift	0.6	X	X
Subsidence	9.0	X	
Crustal Motion From Earthquakes	6.0	X	
Frost Heave of Bench Marks	0.5	X	
Destroyed Bench Marks	—	X	

- **NAVD 88**
  - North American Vertical Datum of 1988
  - One height held fixed at "Father Point" (Rimouski, Canada)
  - …height chosen was to minimize 1929/1988 differences in USGS maps
  - Thus, the "zero height surface" of NAVD 88 wasn't chosen for its closeness to the geoid (but it was close…few decimeters)

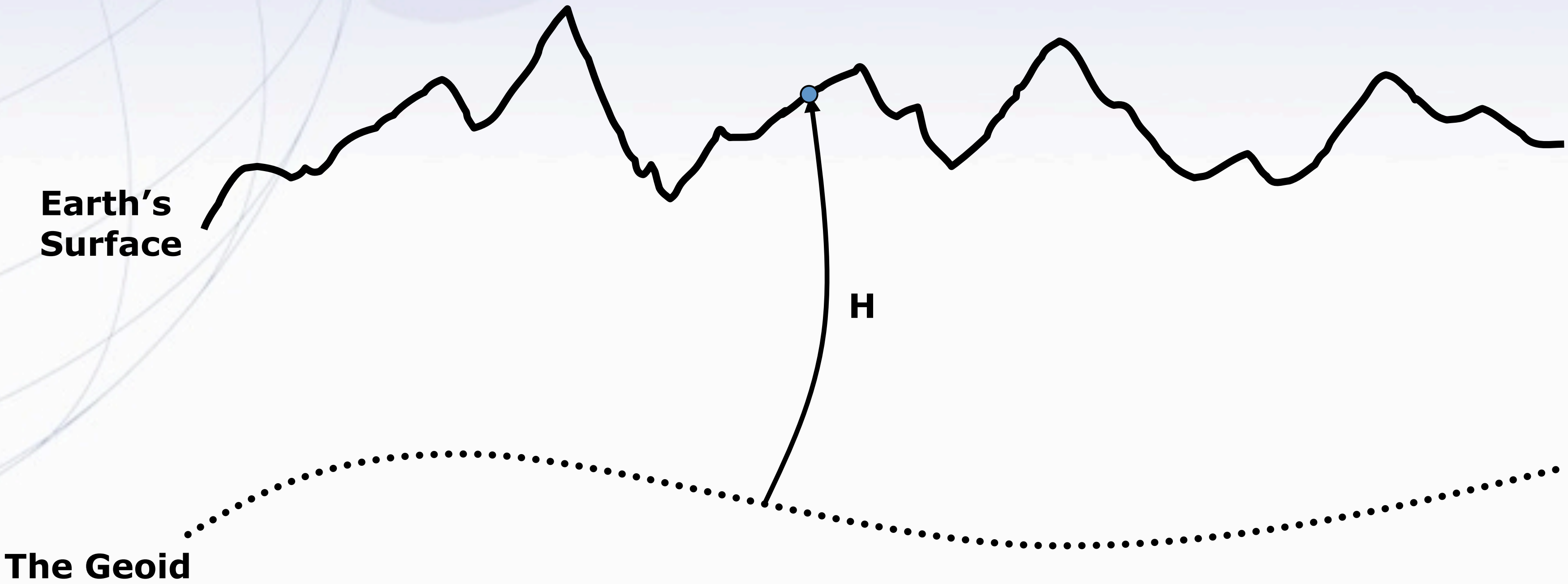
- **NAVD 88** (continued)
  - Use of one fixed height removed local sea level variation problem of NGVD 29
  - Use of one fixed height did open the possibility of unconstrained cross-continent error build up
  - But the H=0 surface of NAVD 88 was supposed to be parallel to the geoid…(close again)

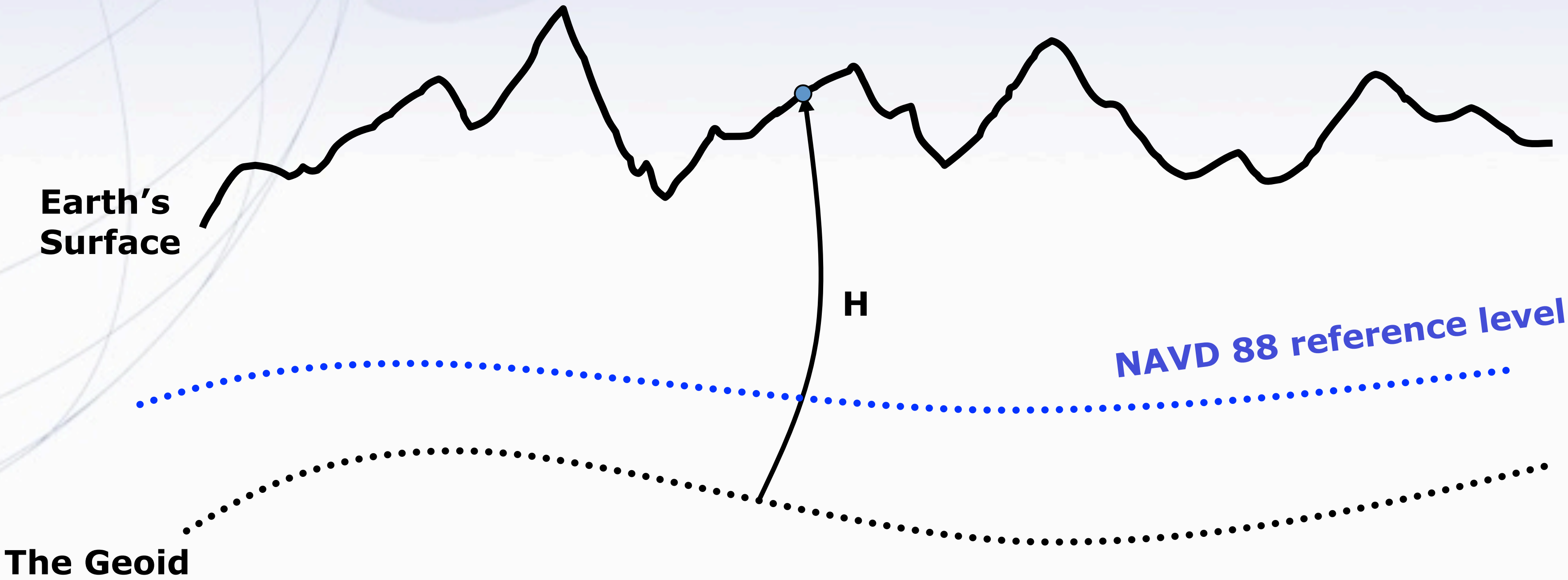
Why isn't NAVD 88 good enough?

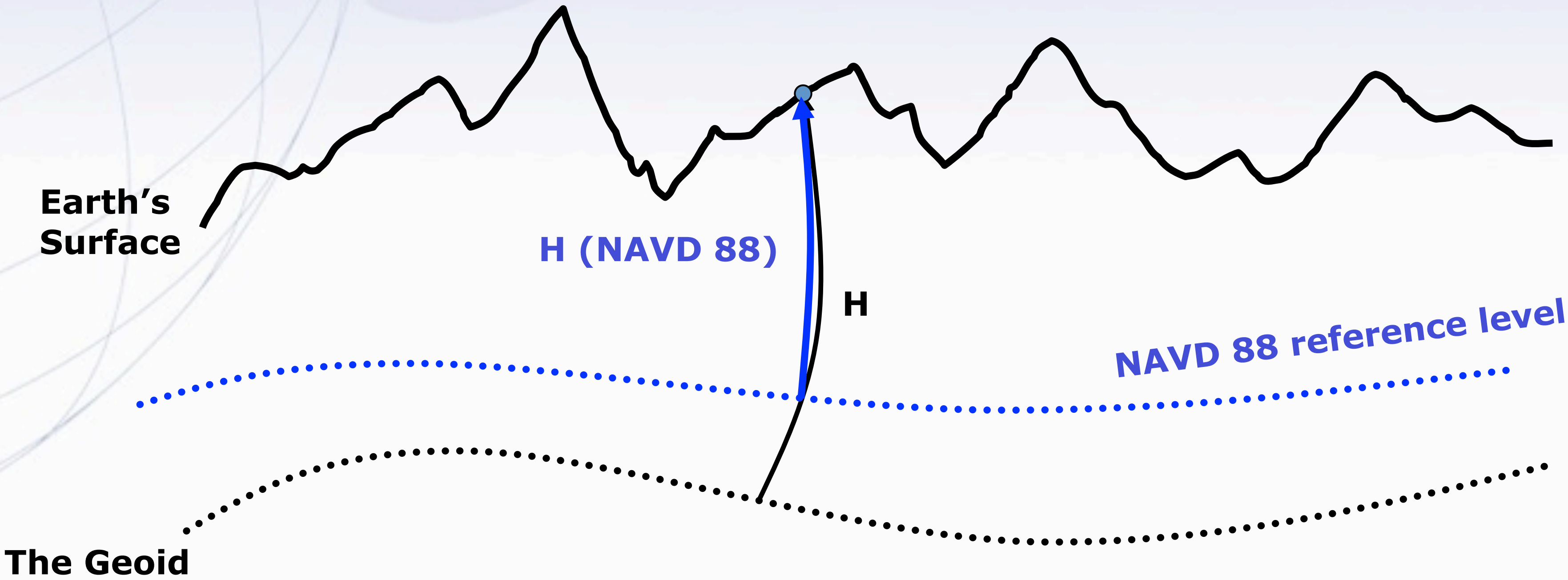
- **NAVD 88 suffers from use of bench marks that:**
  - Are almost never re-checked for movement
  - Disappear by the thousands every year
  - Are not funded for replacement
  - Are not necessarily in convenient places
  - Don't exist in most of Alaska
  - Weren't adopted in Canada
  - Were determined by leveling from a single point, allowing cross-country error build up

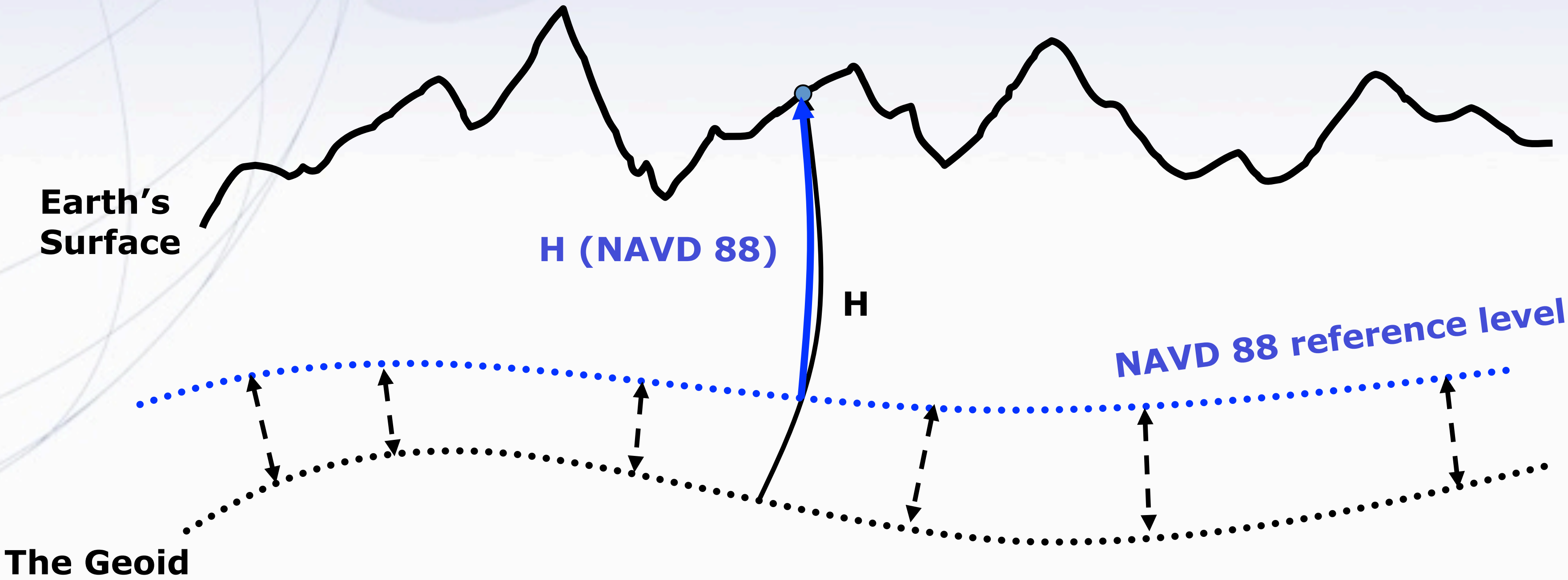
- **NAVD 88 suffers from:**

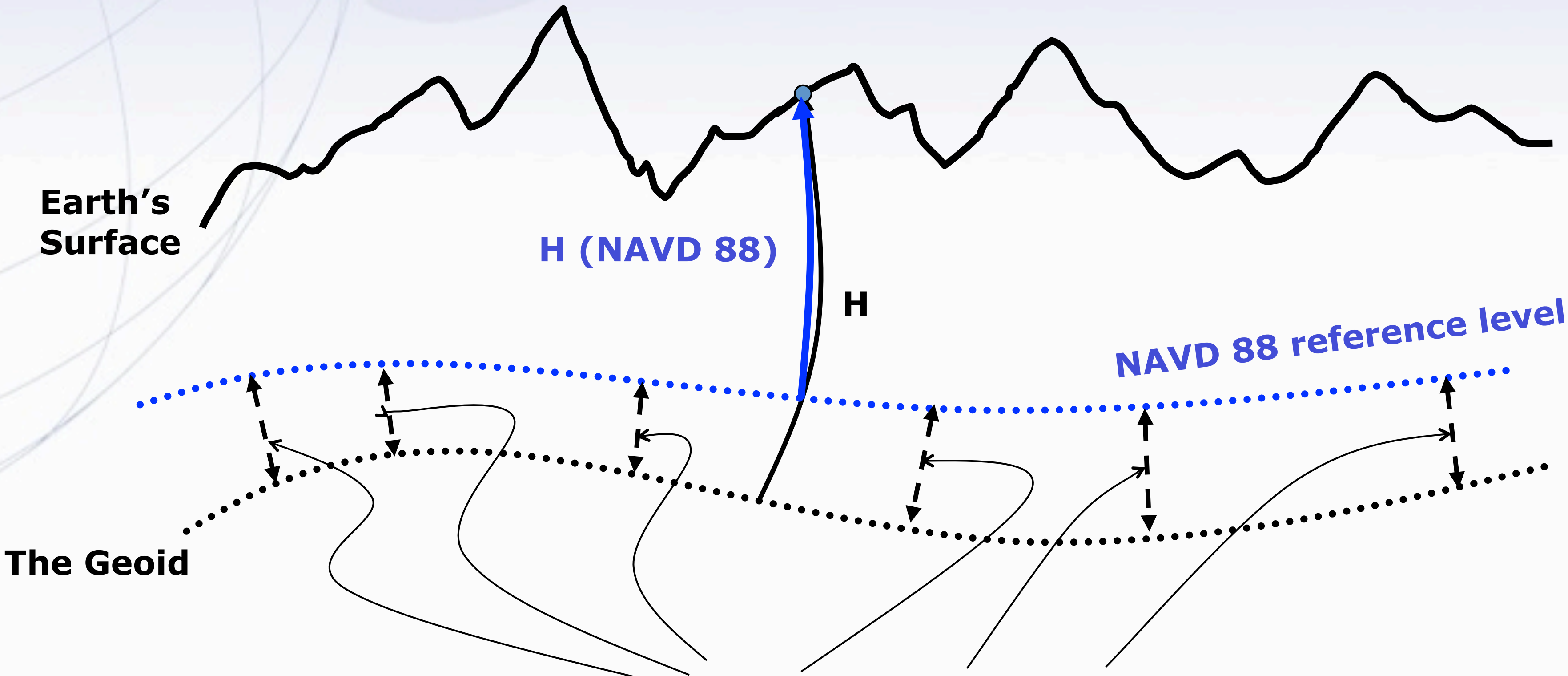
- A zero height surface that:
  - Has been proven to be ~50 cm biased from the latest, best geoid models (GRACE satellite)
  - Has been proven to be ~ 1 meter tilted across CONUS (again, based on the independently computed geoid from the GRACE satellite)


Earth's
Surface
H
The Geoid


Earth's Surface
H
NAVD 88 reference level
The Geoid

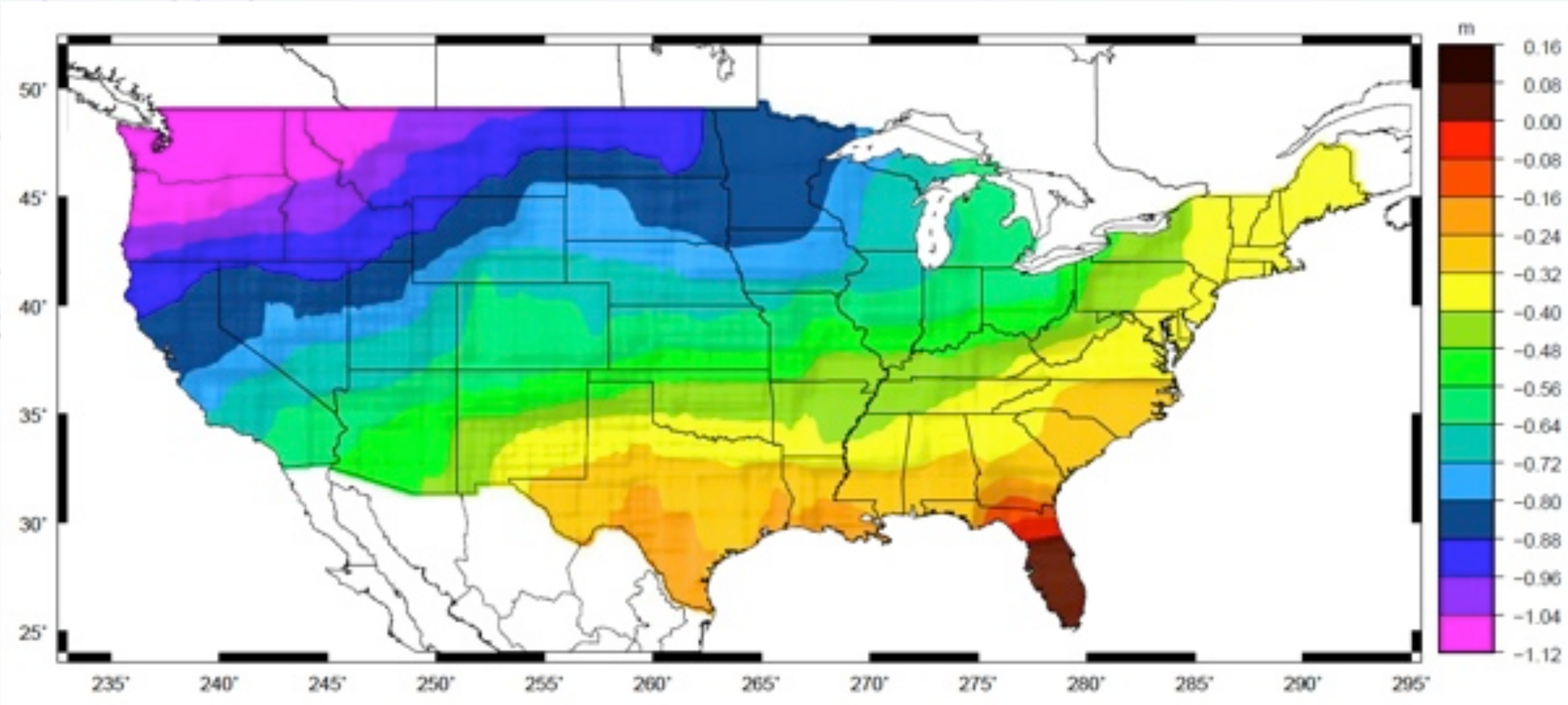

Earth's Surface
H (NAVD 88)
H
NAVD 88 reference level
The Geoid


Earth's Surface
H (NAVD 88)
H
NAVD 88 reference level
The Geoid



Errors in NAVD 88 : ~50 cm average,
100 cm CONUS tilt,
1-2 meters average in Alaska
NO tracking

- **Approximate level of geoid mismatch known to exist in the NAVD 88 zero surface:**



NAVD 88

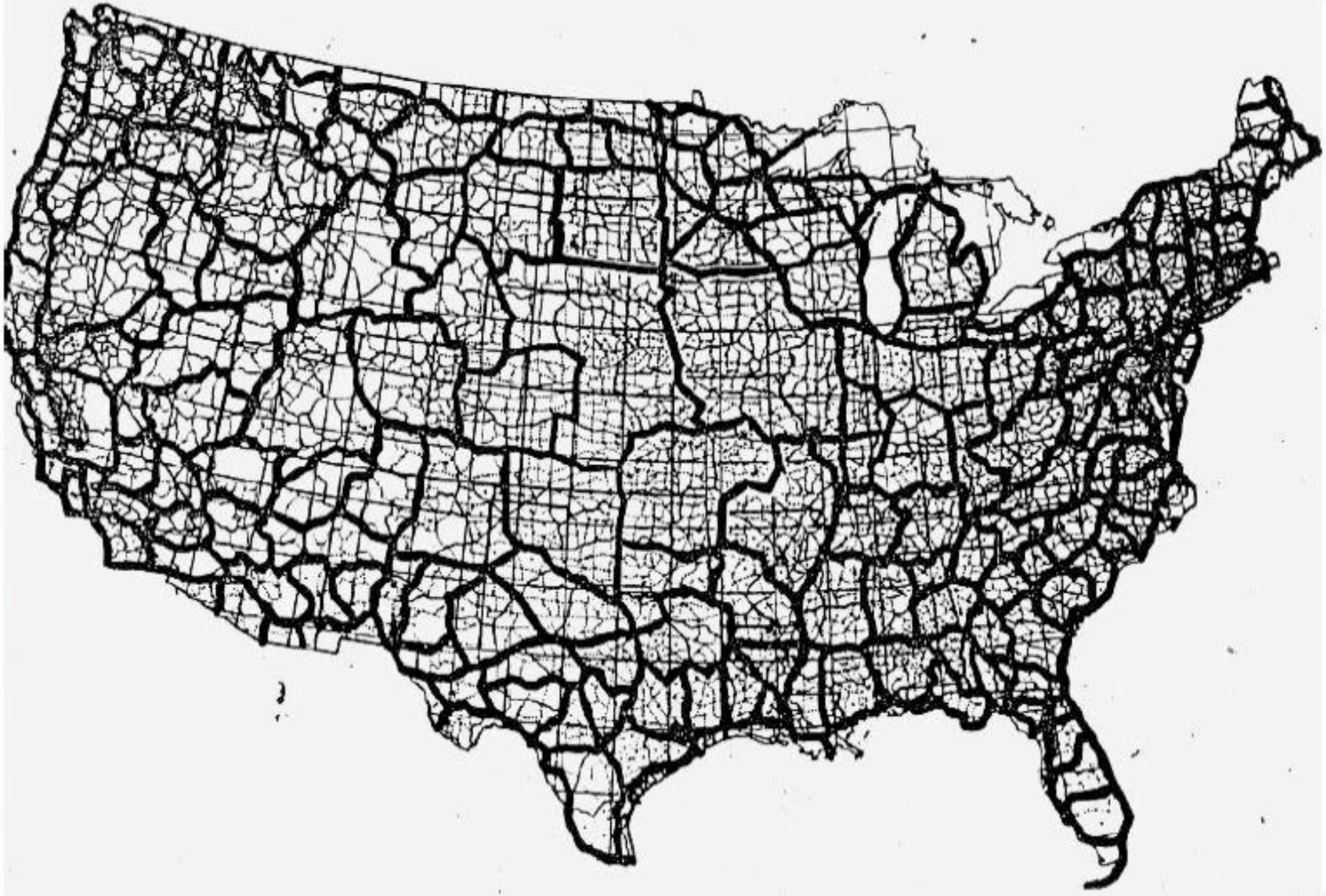
- Datum based on an equipotential surface
- Minimally constrained at one point: Father Point/Rimouski on St. Lawrence Seaway
- 1.3 million kilometers of level data
- Heights determined for 585,000 permanent monuments

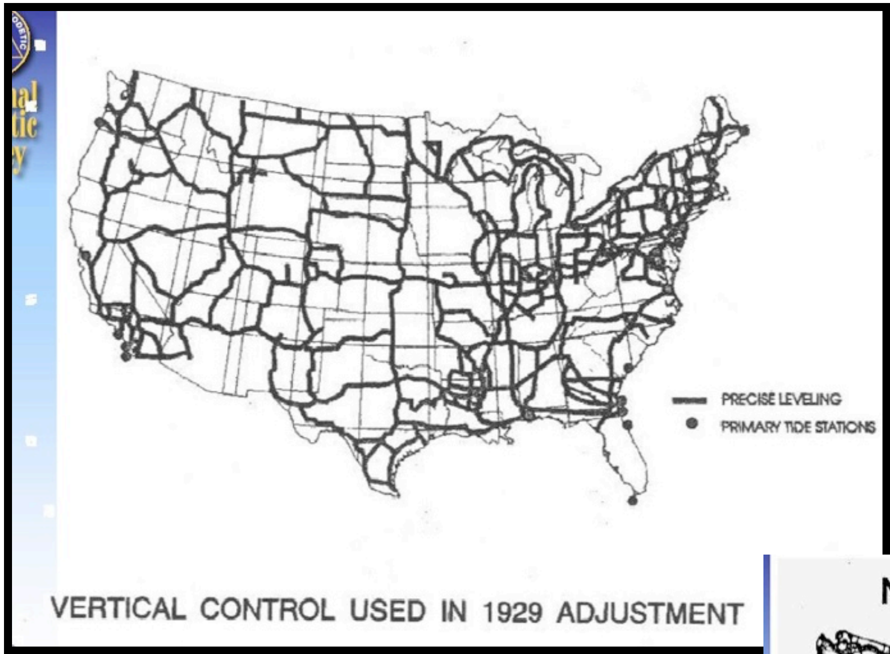
Father Point/Rimouski

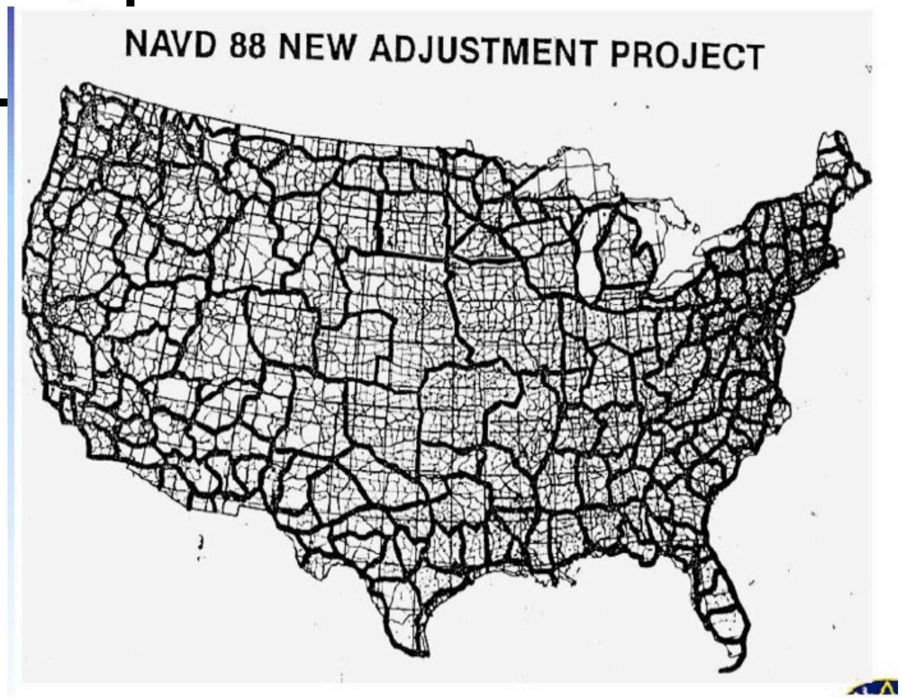
Elements of NAVD 88

- Detected and removed height errors due to blunders
- Minimized effects of systematic errors in leveling data
  - improved procedures better modeling
- **Re-monumentation and new leveling**
- Removal of height discrepancies caused by inconsistent constraints.

NAVD 88 NEW ADJUSTMENT PROJECT




PRECISE LEVELING
PRIMARY TIDE STATIONS
VERTICAL CONTROL USED IN 1929 ADJUSTMENT


NAVD 88 NEW ADJUSTMENT PROJECT

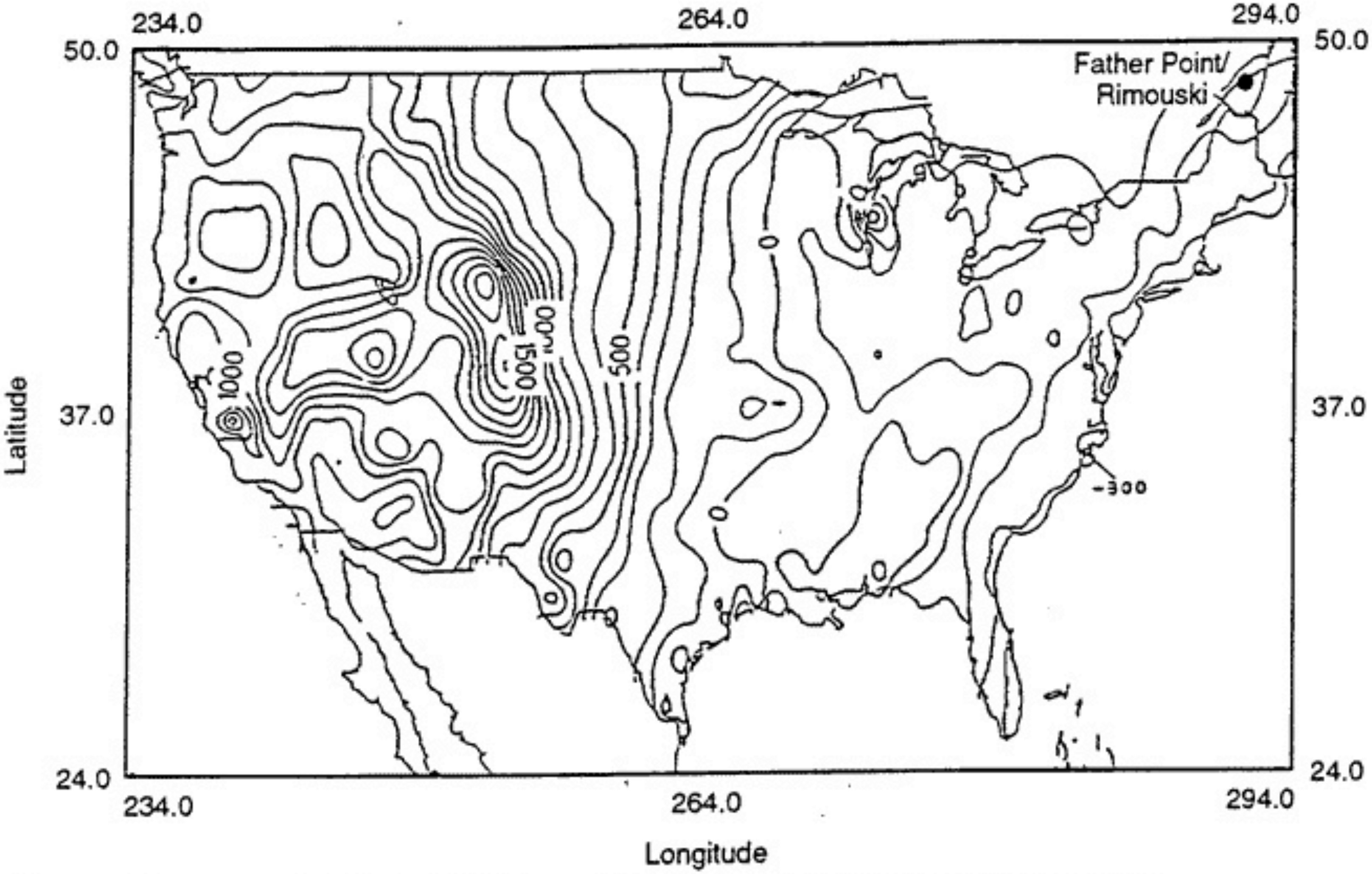


Figure 4. Contour map depicting height differences between NAVD 88 and NGVD 29 (units = mm).

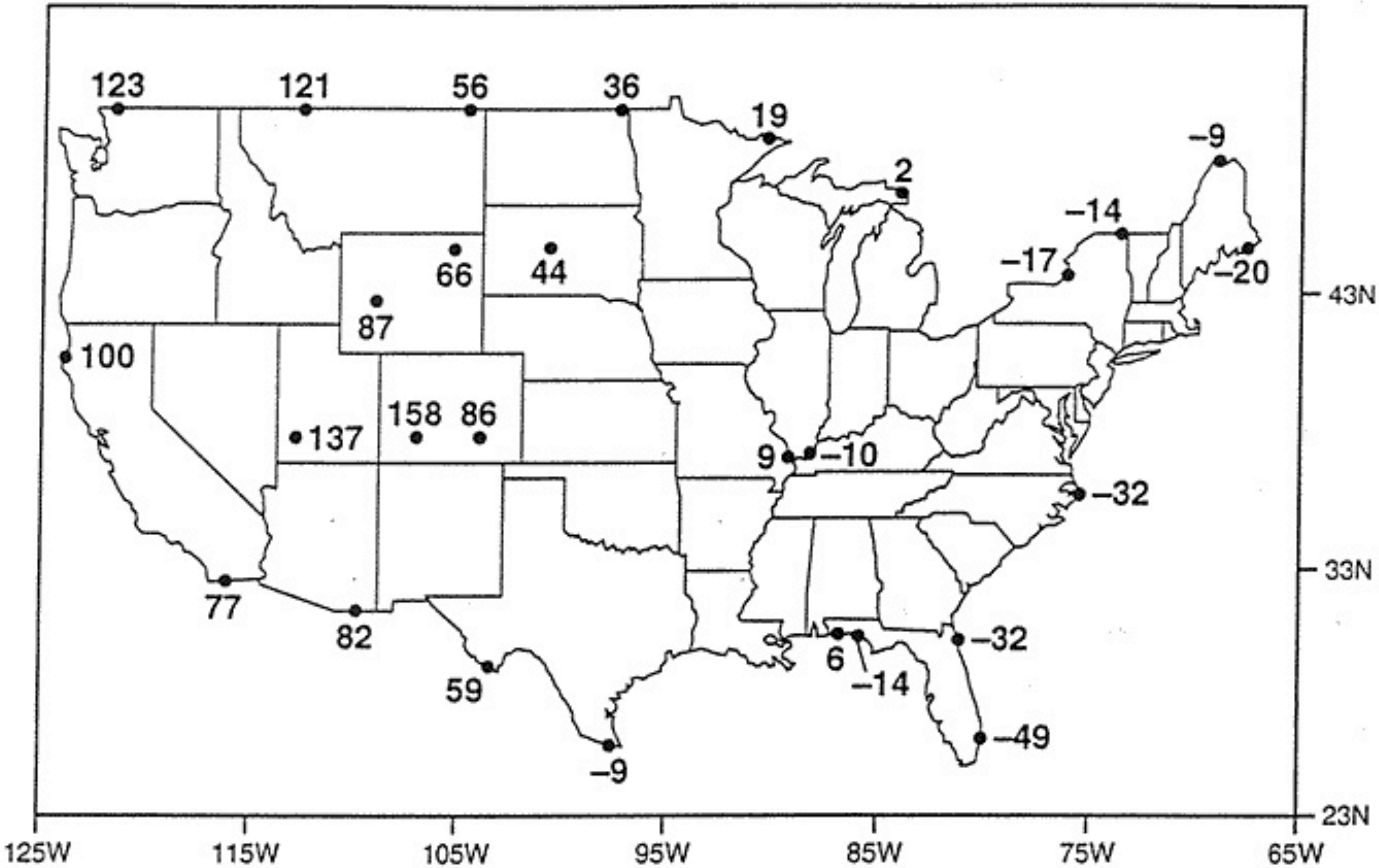
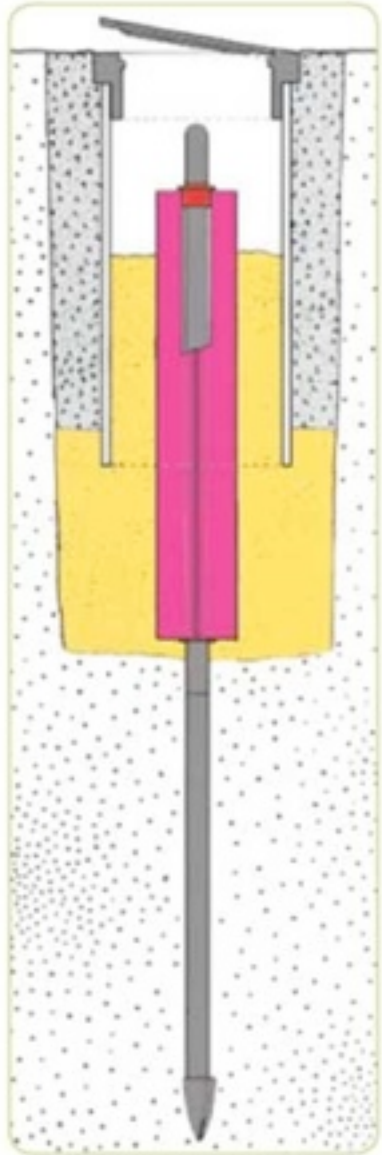
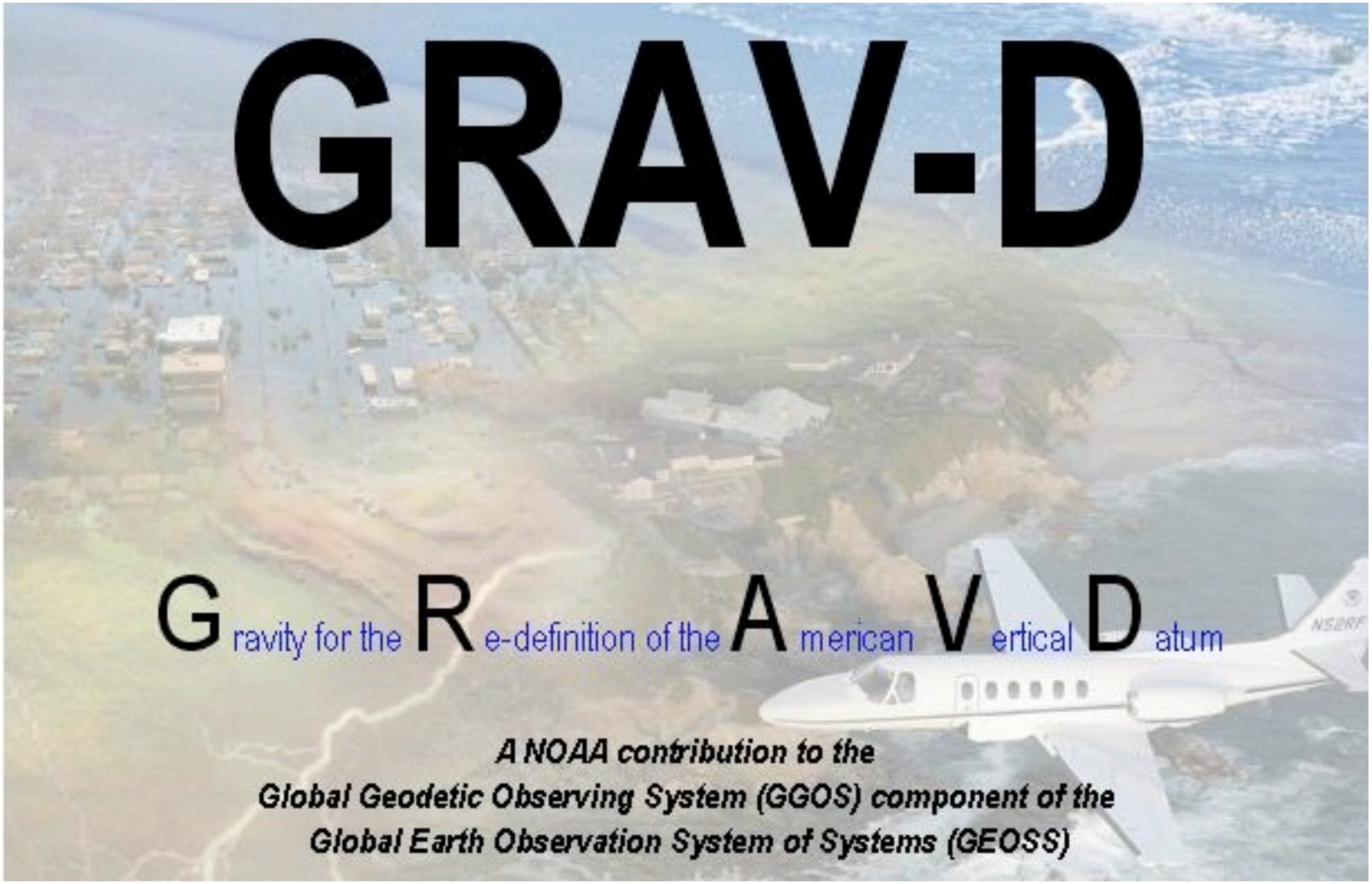


Figure 12. Height differences between NAVD 88 and NGVD 29 in conterminous United States (units = cm).

```
AH0277 ***********************************************************************
AH0277  DESIGNATION -  R 587
AH0277  PID         -  AH0277
AH0277  STATE/COUNTY-  TX/SAN PATRICIO
AH0277  USGS QUAD   -  GREGORY (1975)
AH0277
AH0277                        *CURRENT SURVEY CONTROL
AH0277
AH0277* NAD 83(1986)-  27 53 27.      (N)     097 20 35.      (W)     SCALED
AH0277* NAVD 88     -        10.664   (meters)       34.99    (feet)  ADJUSTED
AH0277
AH0277  GEOID HEIGHT-         -26.68  (meters)                        GEOID03
AH0277  DYNAMIC HT  -          10.648 (meters)       34.93    (feet)  COMP
AH0277  MODELED GRAV-     979,134.4   (mgal)                          NAVD 88
AH0277
AH0277  VERT ORDER  -  FIRST     CLASS II
AH0277
AH0277.The horizontal coordinates were scaled from a topographic map and have
AH0277.an estimated accuracy of +/- 6 seconds.
AH0277
AH0277.The orthometric height was determined by differential leveling
AH0277.and adjusted in June 1991.
AH0277
AH0277.The geoid height was determined by GEOID03.
AH0277
AH0277.The dynamic height is computed by dividing the NAVD 88
AH0277.geopotential number by the normal gravity value computed on the
AH0277.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
AH0277.degrees latitude (g = 980.6199 gals.).
AH0277
AH0277.The modeled gravity was interpolated from observed gravity values.
AH0277
AH0277;                  North         East     Units  Estimated Accuracy
AH0277;SPC TX S     - 5,246,950.     413,910.     MT  (+/- 180 meters Scaled)
AH0277
AH0277                       *SUPERSEDED SURVEY CONTROL
AH0277
AH0277  NGVD 29 (??/??/92)    10.854  (m)            35.61    (f) ADJ UNCH    1 2
```


U.S. COAST & GEODETIC SURVEY
ELEV.
FEET
$250 FINE OR IMPRISONMENT
J 270
1935
FOR DISTURBING THIS MARK
ABOVE MEAN SEA LEVEL
BENCH MARK





New vertical datum to be based on h (ellipsoid heights) and N (gravimetric geoid model).

Remember: **h – H – N = 0 plus errors**

Vertical Datum Transformations

- **First choice: Estimate heights using original leveling data in least squares**
- Second choice: Rigorous transformation using datum conversion correctors estimated by adjustment constraints and differences
- Third option: VERTCON

Linear Regression

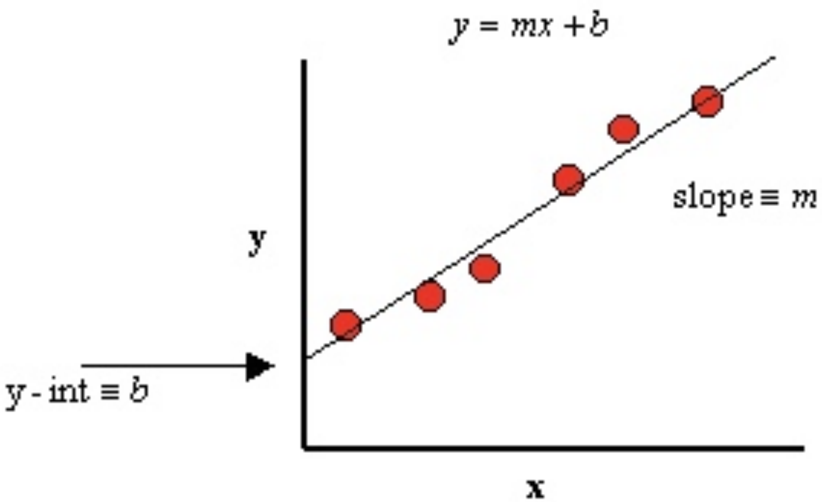
- Linear regression attempts to model the relationship between two variables by fitting a linear equation to observed data.
- A linear regression line has an equation of the form $\boldsymbol{Y = mX + b}$, where $\boldsymbol{X}$ is the explanatory variable and $\boldsymbol{Y}$ is the dependent variable. The slope of the line is **m**, and **b** is the intercept (the value of $\boldsymbol{y}$ when $\boldsymbol{x} = 0$).

x	y
1.0	2.6
2.3	2.8
3.1	3.1
4.8	4.7
5.6	5.1
6.3	5.3

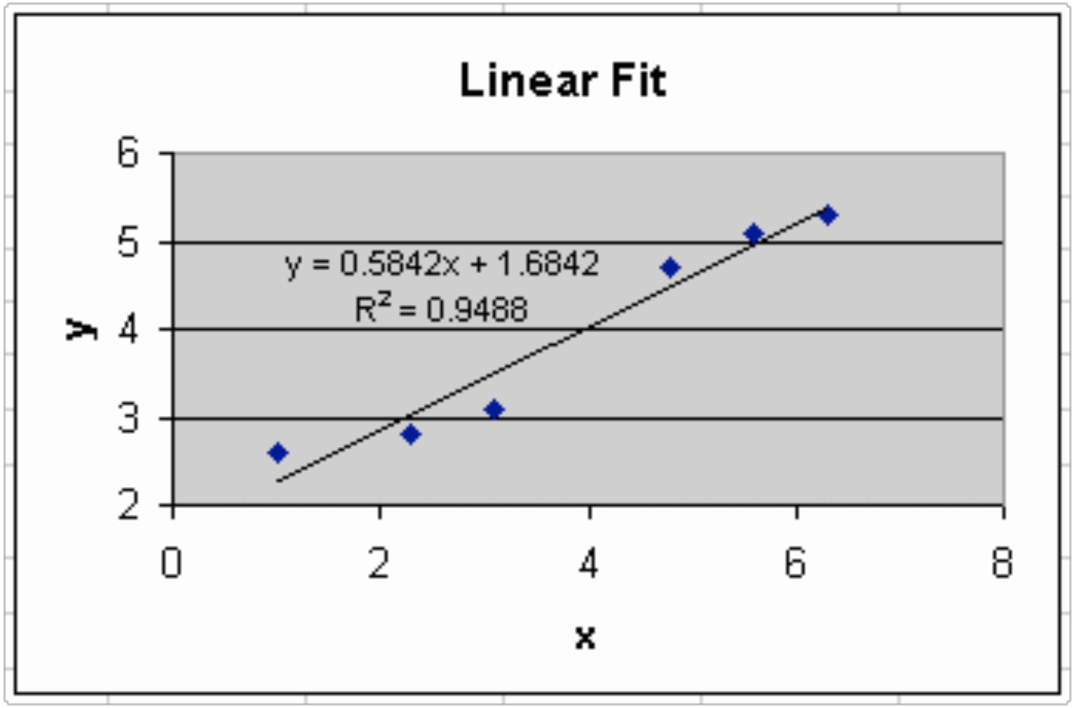
$$m = \frac{n\sum(xy) - \sum x \sum y}{n\sum(x^2) - \left(\sum x\right)^2}$$

$$b = \frac{\sum y - m\sum x}{n}$$

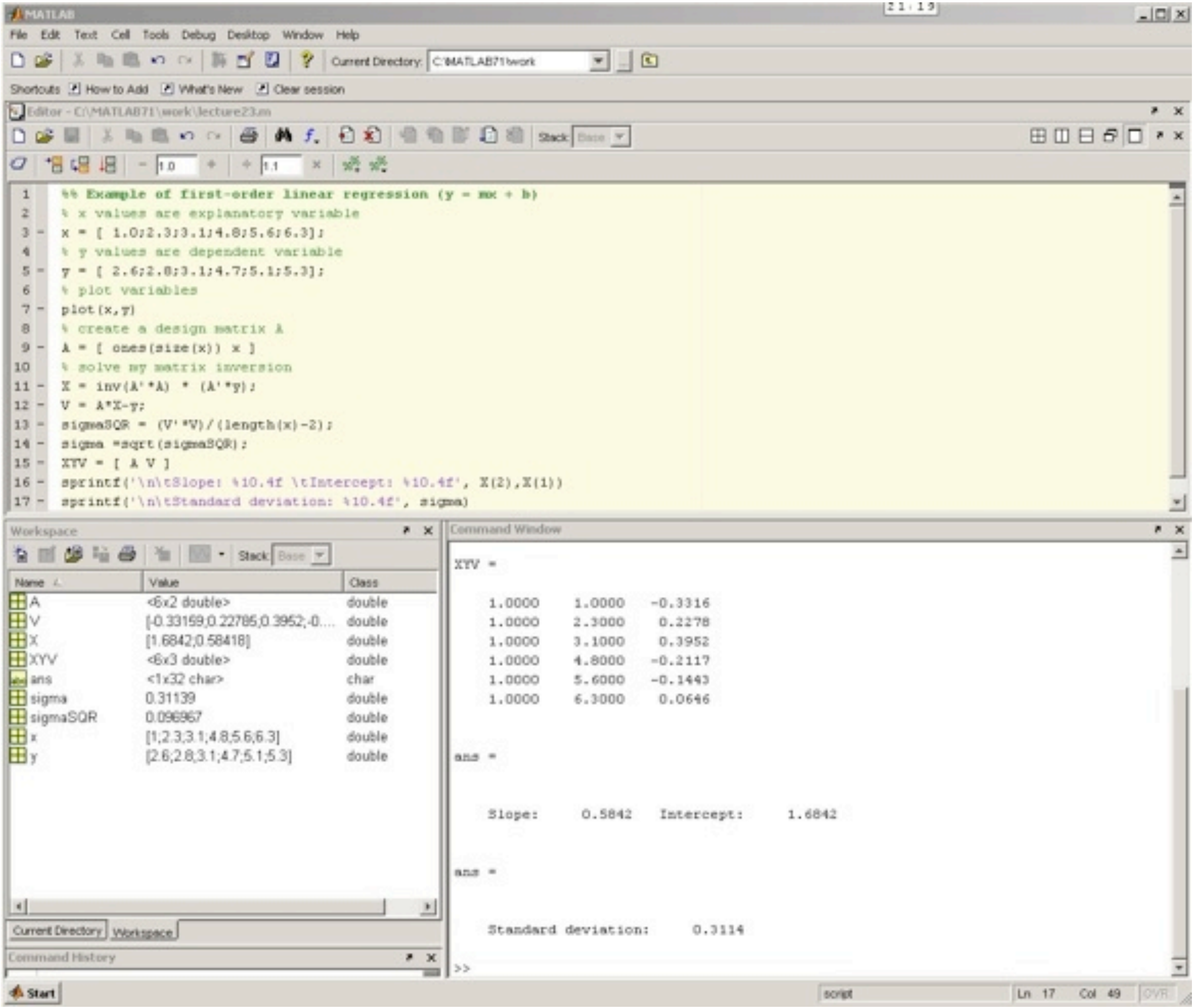
$$r = \frac{n\sum(xy) - \sum x \sum y}{\sqrt{\left[n\sum(x^2) - \left(\sum x\right)^2\right]\left[n\sum(y^2) - \left(\sum y\right)^2\right]}}$$

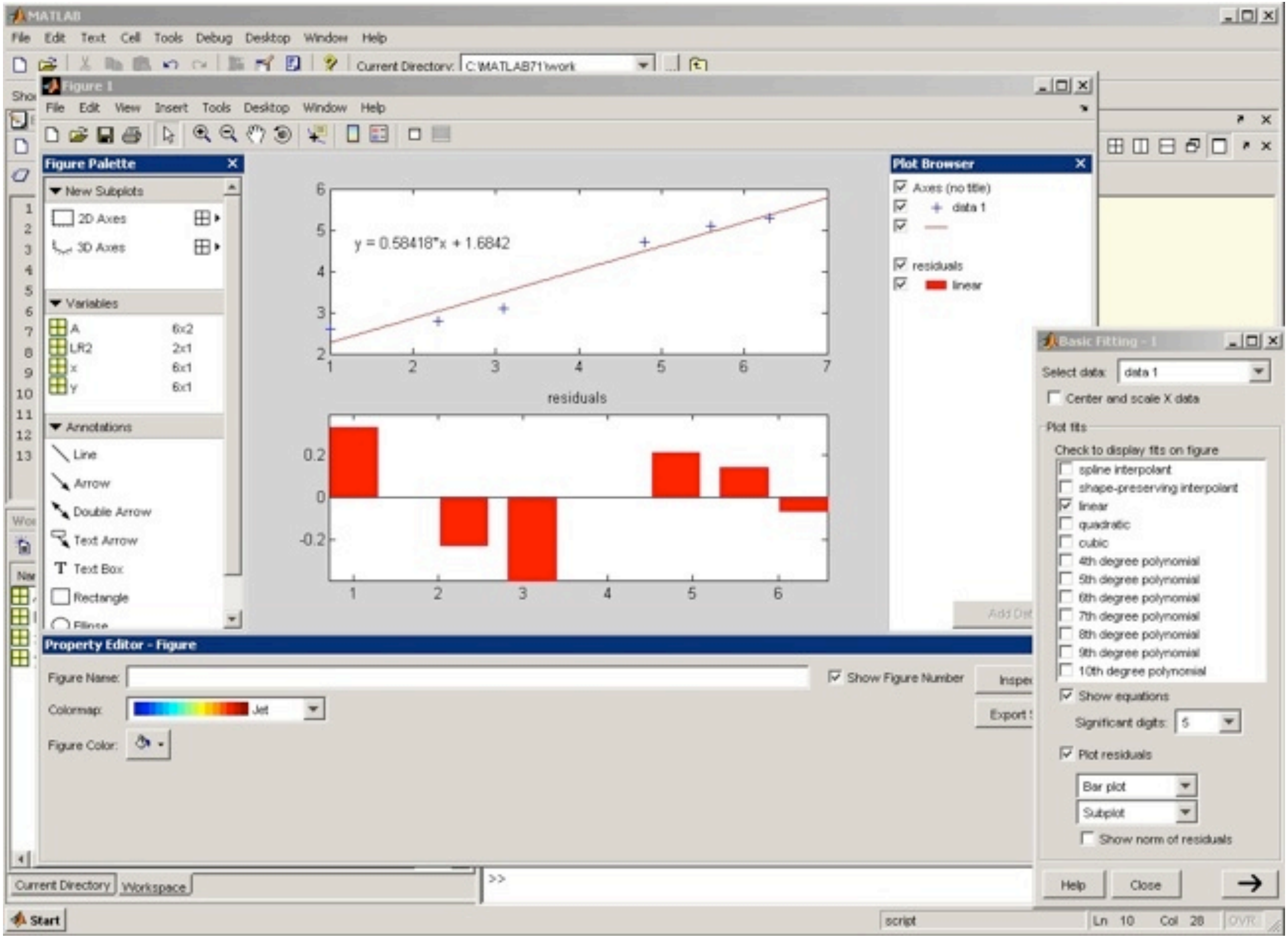


Results in Excel



http://phoenix.phys.clemson.edu/tutorials/excel/regression.html


MATLAB
File Edit Text Cell Tools Debug Desktop Window Help
Current Directory: C:\MATLAB71\work
Shortcuts How to Add What's New Clear session
Editor - C:\MATLAB71\work\lecture23.m
%% Example of first-order linear regression (y = mx + b)
% x values are explanatory variable
x = [1.0;2.3;3.1;4.8;5.6;6.3];
% y values are dependent variable
y = [2.6;2.8;3.1;4.7;5.1;5.3];
% plot variables
plot(x,y)
% create a design matrix A
A = [ones(size(x)) x]
% solve my matrix inversion
X = inv(A'*A) * (A'*y);
V = A*X-y;
sigmaSQR = (V'*V)/(length(x)-2);
sigma =sqrt(sigmaSQR);
XYV = [A V]
sprintf('\n\tSlope: %10.4f \tIntercept: %10.4f', X(2),X(1))
sprintf('\n\tStandard deviation: %10.4f', sigma)
Workspace
Name Value Class
A <6x2 double> double
V [-0.33159;0.22785;0.3952;-0.... double
X [1.6842;0.58418] double
XYV <6x3 double> double
ans <1x32 char> char
sigma 0.31139 double
sigmaSQR 0.096967 double
x [1;2.3;3.1;4.8;5.6;6.3] double
y [2.6;2.8;3.1;4.7;5.1;5.3] double
Current Directory Workspace
Command History
Command Window
XYV =
1.0000 1.0000 -0.3316
1.0000 2.3000 0.2278
1.0000 3.1000 0.3952
1.0000 4.8000 -0.2117
1.0000 5.6000 -0.1443
1.0000 6.3000 0.0646
ans =
Slope: 0.5842 Intercept: 1.6842
ans =
Standard deviation: 0.3114
>>
Start
script Ln 17 Col 49 OVR


MATLAB
File Edit Text Cell Tools Debug Desktop Window Help
Current Directory: C:\MATLAB71\work
Figure 1
File Edit View Insert Tools Desktop Window Help
Figure Palette
New Subplots
2D Axes
3D Axes
Variables
A 6x2
LR2 2x1
x 6x1
y 6x1
Annotations
Line
Arrow
Double Arrow
Text Arrow
Text Box
Rectangle
y = 0.58418*x + 1.6842
residuals
Plot Browser
Axes (no title)
data 1
residuals
linear
Basic Fitting - 1
Select data: data 1
Center and scale X data
Plot fits
Check to display fits on figure
spline interpolant
shape-preserving interpolant
linear
quadratic
cubic
4th degree polynomial
5th degree polynomial
6th degree polynomial
7th degree polynomial
8th degree polynomial
9th degree polynomial
10th degree polynomial
Show equations
Significant digits: 5
Plot residuals
Bar plot
Subplot
Show norm of residuals
Help
Close
Property Editor - Figure
Figure Name:
Show Figure Number
Colormap: Jet
Figure Color:
Current Directory
Workspace
Start
script
Ln 10 Col 28 OVR

ITRF 00

HOUSTON RRP2 (TXHU), TEXAS

Retrieved from NGS DataBase on 03/17/03 at 14:14:49.

Antenna Reference Point(ARP): HOUSTON RRP2 CORS ARP

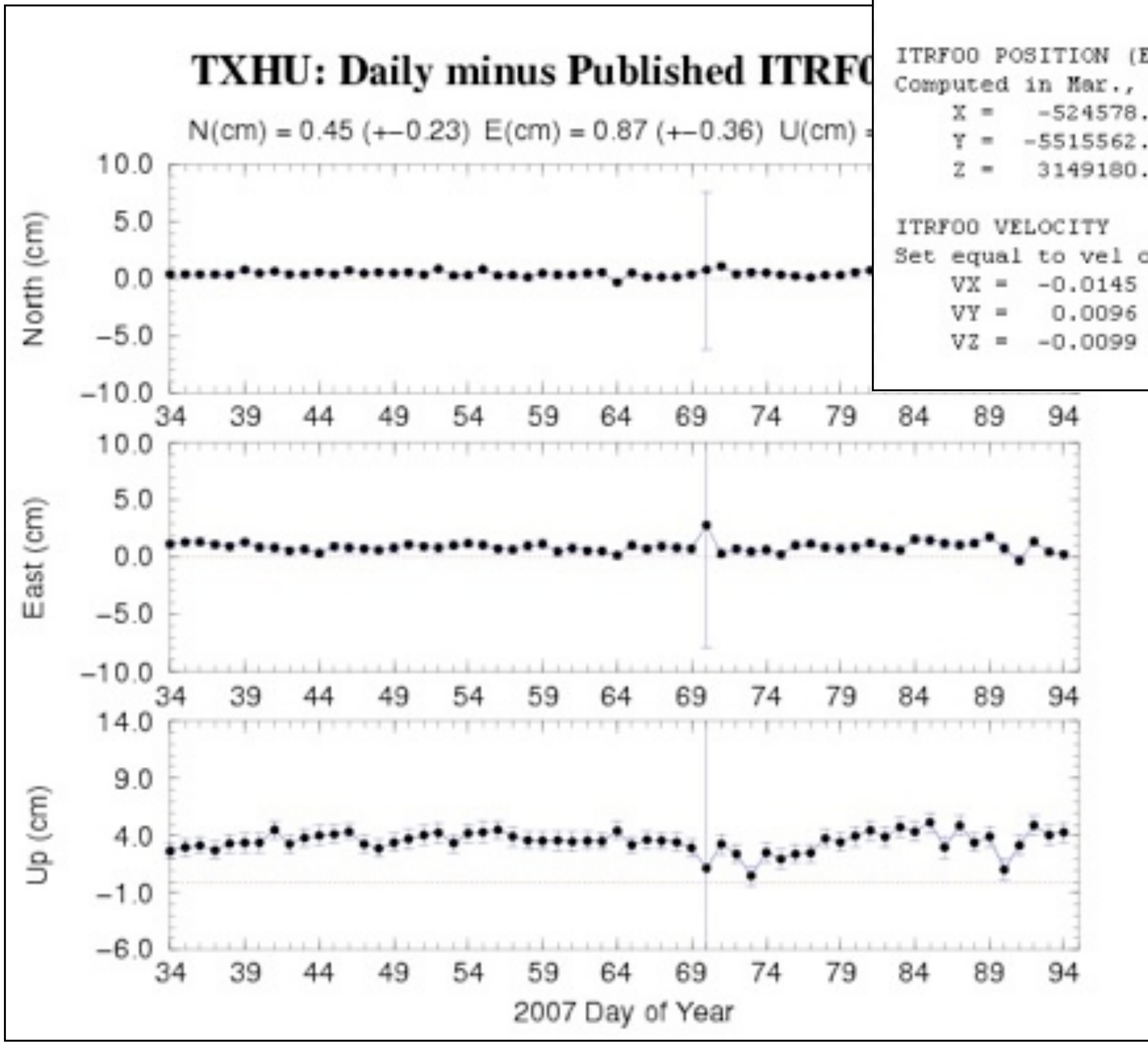
PID = DF4379

ITRF00 POSITION (EPOCH 1997.0)
Computed in Mar., 2003 using 13 days of data.

X =	-524578.515 m	latitude =	29 46 45.91022 N
Y =	-5515562.145 m	longitude =	095 25 58.76631 W
Z =	3149180.614 m	ellipsoid height =	12.001 m

ITRF00 VELOCITY
Set equal to vel of hous Mar., 2003.

VX =	-0.0145 m/yr	northward =	-0.0045 m/yr
VY =	0.0096 m/yr	eastward =	-0.0153 m/yr
VZ =	-0.0099 m/yr	upward =	-0.0120 m/yr



	A	B	C	D	E	F	G	H	I
1									
2	**Segment**	**DA**	**DH**	**n**	**Δ (m)**	**DA*Δ (m2)**	**DA(obs)**	**DA^2**	**V(m)**
3	150-300	149.9929	149.9899	1	0.0030	0.44997870	149.9929	22497.87005	-0.0007
4	300-150	149.9929	149.9905	1	0.0024	0.35998296	149.9929	22497.87005	-0.0013
5	150-600	449.9990	449.9916	1	0.0074	3.32999260	449.999	202499.1	-0.0004
6	600-150	449.9990	449.9849	1	0.0141	6.34498590	449.999	202499.1	0.0063
7	150-1800	1649.9959	1649.9600	1	0.0359	59.23485281	1649.9959	2722486.47	0.0119
8	1800-150	1649.9959	1649.9728	1	0.0231	38.11490529	1649.9959	2722486.47	-0.0009
9	300-600	300.0061	300.0003	1	0.0058	1.74003538	300.0061	90003.66004	0.0001
10	600-300	300.0061	299.9984	1	0.0077	2.31004697	300.0061	90003.66004	0.0020
11	300-1800	1500.0030	1499.9739	1	0.0291	43.65008730	1500.003	2250009	0.0071
12	1800-300	1500.0030	1499.9906	1	0.0124	18.60003720	1500.003	2250009	-0.0096
13	600-1800	1199.9969	1199.9866	1	0.0103	12.35996807	1199.9969	1439992.56	-0.0076
14	1800-600	1199.9969	1199.9858	1	0.0111	13.31996559	1199.9969	1439992.56	-0.0068
15			**sums**	**12**	**0.1623**	**199.81483877**	**10499.9876**	**13454977.32**	**0.0000**
16							**110249739.6**	**sumDA(obs)^2**	
17	**DA**	published							
18	**DH**	observed			**scale**	**0.00001354**	**13.5**	**ppm**	
19	**V(m)**	variance			**constant**	**0.00167330**	**0.0017**	**meters**	
20									

Rod Calibration

CBI level = 333704

Calibration Report

Invar rod (type, No.):

GPCL 3 24304 S/N

No. of graduations measured:

6 - 345

Contract:

04-143-466231

Date :

19.06.04

0.0060

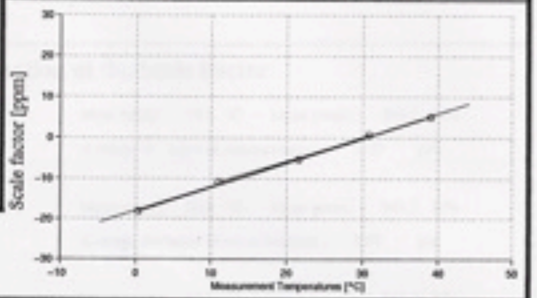
Coefficient of expansion:

α_T = 0.60 ± 0.01 ppm/°C

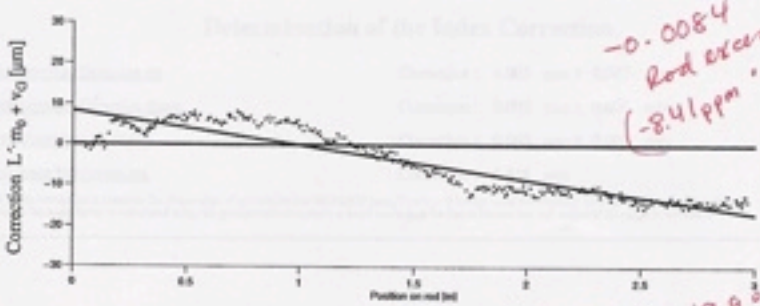
Determination of the coefficient of expansion

Horizontal calibration position

Measurement cycle: 30 → 0 → 20 → 40 → 10 [°C]



Determination of the scale factor **Vertical calibration position**



-0.0084
Rod excess
(-8.41ppm · 1000)

19.9°C
Standardization temperature

Scale factor: m_0 = 8.41 ± 1.07 ppm at T_0 = 19.9 °C

Length adjustment from the vertical calibration (position of use)

$$L = l^0 + L'[1 + (m_0 + \alpha_T(T - T_0)) \cdot 10^{-6}] + v_G$$

l^0 = +0.019 ± 0.005 mm

v_G = +0.008 mm

$l^0 = l^0_K + v_K$

l^0_K = +0.003 ± 0.005 mm

v_K = +0.016 mm

L'[m] = observed rod length v_G [m] = graduation correction

T[°C] = temperature

l^0 [m] = index correction (l^0_K[m] = index correction of reference bar, v_K[m] = reference bar correction)

Technical specialist: [signature]   Munich, 25.06.2004 (2004 06 25)

Laboratory director: [signature]   Institute director: i.V. [signature]

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Arcisstraße 21, 80290 München, Tel.: 089/289-22850, Fax: 089 /289-23967

Laboratory of Metrology and Geodesy

PEPS, Laval University Tel.: (418)656-7082
Sainte-Foy, Qc, Canada FAX: (418)656-2014
G1K 7P4 France.Plante@SsG.ULaval.Ca

Certificate of Calibration no. 9813

For: Maryland Department of Transportation
Invar Staff Leica no. 28793

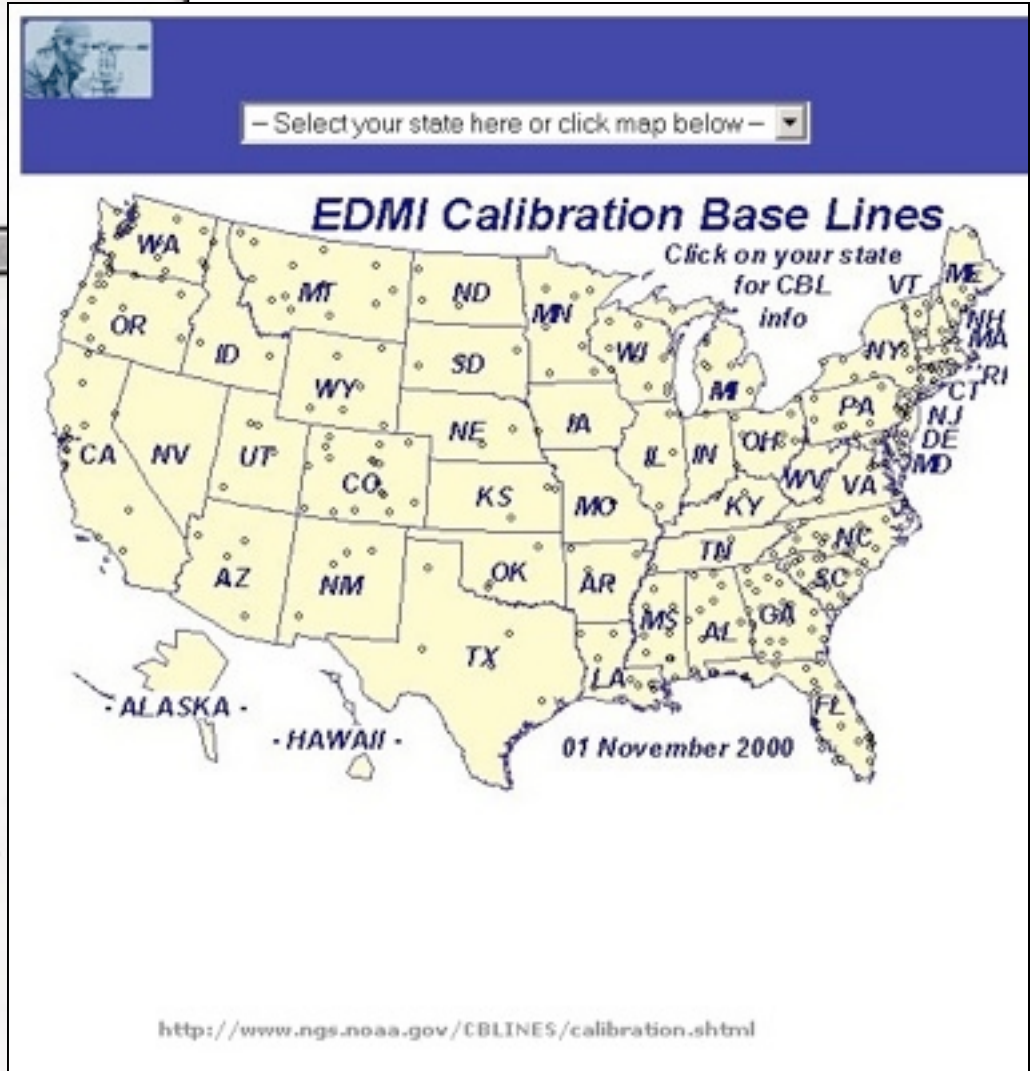
Face-Value (mm)	Observations (mm)
118.4625	118.44
306.7875	306.77
501.1875	501.17
695.5875	695.57
898.0875	898.07
1090.4625	1090.45
1307.1375	1307.12
1495.4625	1495.44
1691.8875	1691.87
1890.3375	1890.32
2094.8625	2094.84
2287.2375	2287.22
2507.9625	2507.94
2694.2625	2694.24
2917.0125	2916.99

Notes:
Temperature of calibration: 20° ± 0.7° C.
Instrument used for control: Interferometer Hewlett-Packard model 5508A (s.n.: 2230A00169).
Resolution of interferometer: 0.00001 mm.
Estimed precision of results: 0.022 mm.

Novembre 26 1998

Director of Laboratory of Metrology and Geodesy

France Plante, M.Sc., a.-g.



Two-Plane Method of Interpolating Heights (Problem 8.3)

- We can approximate the shift at an unknown point (when observations are unavailable) using least squares methods.
  - Need minimum of four points with known elevations in both vertical datums.
  - Need plane coordinates for all points.
  - Calculates rotation angles in both planes (N-S and E-W) as well as the vertical shift.

A Matlab-based solution is provided on the class web page.

Problem 8.3 in text

Benchmark	NGVD 29 Height ft.	NAVD 88 m	Northing	Easting
Q 547	4088.82	1247.360	60,320	1,395,020
A 15	4181.56	1275.636	60,560	1,399,870
AIRPORT 2	4085.32	1246.314	56,300	1,397,560
NORTH BASE	4191.80	1278.748	57,867	1,401,028
T 547	4104.04	Unknown	58,670	1,397,840

Function model

- $(NAVD88_i - NGVD29_i) = \alpha_E(N_i - N_0) + \alpha_N(E_i - E_0) + t_Z$
- Where we compute the following (all values in meters):
  - $NAVD88_i - NGVD29_i$ = difference in heights
  - $N_i - N_0$ = is difference of each North coordinate of known points from centroid
  - $E_i - E_0$ = is difference of each East coordinate of known points from centroid

Solving Problem

- Determine the mean value (centroid) for N and E coordinates (use known points only)
  - N_0: 58762 E_0: 1398370 (**wrong in text**)
- Determine NAVD 88 - NGVD 29 for points with values in both systems. Note signs!

Δ Q 547 = 1.085

Δ A 15 = 1.094

Δ AIRPORT 2: = 1.106

Δ NORTH BASE = 1.085

Compute differences from centroid

Station	Difference in N	Difference in E
Q 547	1558	-3350
A 15	1798	1500
AIRPORT 2	-2462	-810
NORTH BASE	-895	2658

Compute parameters

- B the design matrix consists of three columns:
  - Col.1: difference in Northings from centroid
  - Col.2: difference in Eastings from centroid
  - Col.3: all ones
- F the observation matrix
  - Vector of height differences
- Parameters are computed by the method of least squares: $(B^T B)^{-1} B^T f$

$$\text{Datum Shift}_i = (\text{NAVD88}_i - \text{NGVD29}_i) = \alpha_E (N_i - N_0) + \alpha_N (E_i - E_0) + t_Z$$

One equation may be written for each of the four known benchmarks (after simplifying):

Q547: $1.085 \text{ m} = \alpha_E (1558) + \alpha_N (-3350) + t_Z$
A15: $1.094 \text{ m} = \alpha_E (1798) + \alpha_N (1501) + t_Z$
AIRPORT 2: $1.106 \text{ m} = \alpha_E (-2462) + \alpha_N (-810) + t_Z$
NORTH BASE: $1.085 \text{ m} = \alpha_E (-895) + \alpha_N (2659) + t_Z$

The above equations are of the form $\mathbf{f} = \mathbf{B}\Delta + \mathbf{v}$ where **f** represents the vector of observations, **B** represents the matrix of parameter coefficients, Δ represents the vector of parameters and **v** represents the vector of observation residuals. The parameters for this system of equations may be estimated using unweighted least squares by the adjustment of indirect observations (Mikhail, 1976). The vector of parameters is computed using $\Delta = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{f}$.

$$\mathbf{B} = \begin{bmatrix} 1558 & -3350 & 1 \\ 1798 & 1501 & 1 \\ -2462 & -810 & 1 \\ -895 & 2659 & 1 \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 1.085 \\ 1.094 \\ 1.106 \\ 1.085 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \alpha_E \\ \alpha_N \\ t_Z \end{bmatrix} = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{f} = \begin{bmatrix} -2.96861 \times 10^{-6} \text{ radians} \\ -5.72027 \times 10^{-7} \text{ radians} \\ 1.092 \text{ m} \end{bmatrix}$$

T547: Datum Shift = $(-2.96861 \times 10^{-6})(-92) + (-5.72027 \times 10^{-7})(-530) + 1.092 = 1.093$ m.

$\therefore H_{T547} = 4104.04 \text{ US ft} + 1.093 \text{ m} = 1250.914 \text{ m} + 1.093 \text{ m} = 1252.006 \text{ m}.$

The accuracy of this value may be estimated by evaluation of the vector of observation residuals computed using $\mathbf{v} = \mathbf{f} - \mathbf{B}\Delta$. This vector describes the "fit" between the planes representing the NAVD 88 datum and the NGVD 29 datum.

Applying parameters

- Our matrix inversion solved for rotations in E and N as well as shift in height.
- Compute the shift at our location using our functional model: $\alpha_E(N_i-N_0)+ \alpha_N(E_i-E_0)+t_Z$
  - Result is the magnitude of the shift.
- We calculate the new height by algebraically adding the shift to the height in the old system.

$$\mathbf{B} = \begin{bmatrix} 1558 & -3350 & 1 \\ 1798 & 1501 & 1 \\ -2462 & -810 & 1 \\ -895 & 2659 & 1 \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} 1.085 \\ 1.094 \\ 1.106 \\ 1.085 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \alpha_E \\ \alpha_N \\ t_Z \end{bmatrix} = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{f} = \begin{bmatrix} -2.96861 \times 10^{-6} \text{ radians} \\ -5.72027 \times 10^{-7} \text{ radians} \\ 1.092 \text{ m} \end{bmatrix}$$

T547: Datum Shift = $(-2.96861 \times 10^{-6})(-92) + (-5.72027 \times 10^{-7})(-530) + 1.092 = 1.093$ m.

$\therefore H_{T547}$ = 4104.04 US ft + 1.093 m = 1250.914 m + 1.093 m = 1252.006 m.

The accuracy of this value may be estimated by evaluation of the vector of observation residuals computed using **v** = **f** - **B**Δ. This vector describes the "fit" between the planes representing the NAVD 88 datum and the NGVD 29 datum.

$$\mathbf{v} = \begin{bmatrix} -0.005 \text{ m} \\ 0.008 \text{ m} \\ 0.006 \text{ m} \\ -0.009 \text{ m} \end{bmatrix}$$

We validate the accuracy of our result by computing the variances (V).